VISCOUS EFFECTS IN LOW-DENSITY NOZZLE FLOWS

David L. Whitfield

ARO, Inc.

June 1973

VON KÁRMÁN GAS DYNAMICS FACILITY
ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
ARNOLD AIR FORCE STATION, TENNESSEE

NOTICES

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FOREWORD

The work reported herein was conducted by the Arnold Engineering Development Center (AEDC) under the sponsorship of the Air Force Rocket Propulsion Laboratory (AFRPL), Air Force Systems Command (AFSC), under Program Element 62302F. This work was monitored by Captain Sam Thompson of AFRPL.

The results presented were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of the Arnold Engineering Development Center (AEDC), AFSC, Arnold Air Force Station, Tennessee. The work was conducted from March 9 to November 30, 1972, under ARO Project Nos. VY0202 and VC097. The manuscript was submitted for publication on January 18, 1973.

This technical report has been reviewed and is approved.

JIMMY W. MULLINS　　　　　　　　A. L. COAPMAN
Lt Colonel, USAF　　　　　　　　　Colonel, USAF
Chief Air Force Test Director, VKF　Director of Test
Directorate of Test

ABSTRACT

Viscous effects in low-density nozzle flows were investigated numerically, and comparisons were made with experimental data. The numerical method of Patankar and Spalding was modified to solve the internal laminar boundary-layer equations for two-dimensional flow or axisymmetric flow with or without transverse curvature. A listing is given of the computer code. Boundary-layer displacement thicknesses for typical nozzle geometries and flow conditions are presented. Solutions were obtained for specific conditions corresponding to experimental data. The result is a relatively fast, simple to use numerical procedure, which is shown to give results in good agreement with experimental data.

CONTENTS

APPENDIXES

I. ILLUSTRATIONS

NOMENCLATURE

A	Cross-sectional area of nozzle
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
d^*	Nozzle throat diameter
H	Local total enthalpy
$\overline{H}$	H/H_o
h	Heat-transfer coefficient
k	Thermal conductivity
M	Mach number
$\dot{m}$	Mass flux
Pr	Prandtl number, $\mu c_p/k$
p	Pressure
p_p	Measured pitot pressure
$\overline{p}$	$(\gamma - 1)p/(\gamma p_o)$
q_w	Nozzle wall heat-transfer rate
R	Longitudinal radius of curvature of converging portion of nozzle
$\overline{R}$	R/r^*
Re_{o,r^*}	Nozzle reservoir Reynolds number, $\rho_o(2H_o)^{1/2}r^*/\mu_o$

r	Defined by Eq. (4)
$\bar{r}$	r/r^*
r^*	Nozzle throat radius
T	Temperature
T_{aw}	Adiabatic wall temperature
$\bar{T}$	T/T_0
u	Velocity component in x direction
$\bar{u}$	$u/(H_0)^{1/2}$
v	Velocity component in y direction
$\bar{v}$	$v/(H_0)^{1/2}$
x	Coordinate along nozzle wall
$\bar{x}$	x/r^*
y	Coordinate normal to nozzle wall
$\bar{y}$	y/r^*
z	Coordinate along nozzle axis referenced to the throat
$\bar{z}$	z/r^*
α	Nozzle wall angle
γ	c_p/c_v
δ	Boundary-layer thickness defined as the value of y where $u/u_E = 0.99$
δ^*	Boundary-layer displacement thickness
ζ	Exponent in power law viscosity, $\mu \sim T^\zeta$
η	$\bar{y}/\bar{y}_E$
μ	Viscosity
$\bar{\mu}$	μ/μ_0
ν	Denotes planar or axisymmetric flow in Eq. (23)
ξ	Transformed x coordinate, Eq. (11)
ρ	Mass density
$\bar{\rho}$	ρ/ρ_0

ψ	Stream function
ω	Transformed stream function, Eq. (12)

SUPERSCRIPT

Condition immediately downstream of a normal shock

SUBSCRIPTS

$\mathcal{C}_L$	Nozzle centerline
E	Outer edge of boundary layer
e	Nozzle exit
I	Inner edge of boundary layer
o	Reservoir (total) conditions
w	Nozzle wall

SECTION I
INTRODUCTION

Investigations of viscous effects in low-density gas flows in two-dimensional and axisymmetric channels and nozzles have been conducted during the past few years in support of the design of low-density wind tunnel facilities and small microthrust rockets used for spacecraft attitude control. In addition to these areas of interest, there are two other areas where attention to viscous effects is required. One is that of internal boundary-layer scaling. When the nozzle and/or plume flow of a rocket engine is to be investigated in a wind tunnel or space chamber, it is usually necessary to significantly reduce the size of the nozzle used. To achieve adequate simulation, the model nozzle viscous effects must appropriately simulate those of the actual rocket. A specific example of this problem is found in the laboratory study of rocket exhaust plumes interacting with the free-stream (Ref. 1). The other area of interest is associated with the study of gas dynamic and chemical lasers (Ref. 2). The total laser power output is influenced by the static gas pressure in the optical cavity just downstream of the nozzle exit. Since the nozzles used are small and since they operate at relatively high total temperatures, the nozzle viscous effects significantly influence the nozzle static pressure distribution. Although numerous comparisons with experimental data from low-density wind tunnel nozzles will be presented to check the accuracy of the results, the present investigation was motivated by the current interest in rocket nozzle scaling and the viscous effects on the operation of gas dynamic and chemical laser systems.

Some previous investigations of the design and analysis of low-density wind tunnel nozzles are given in Refs. 3 through 5. The method of Potter and Carden (Ref. 3) was developed to design nozzles for particular test section flow conditions. It is based on an integral technique which uses the similar solutions of Cohen and Reshotko (Ref. 6). Although the work of Potter and Carden (Ref. 3) has proved successful for the design of nozzles to produce desired flow conditions, it is not directly applicable to the analysis of specified nozzle geometry. Also, non-similar solutions presented in Ref. 4 indicate that similarity does not exist in nozzle flows, particularly near the throat, and as a consequence, inaccuracies may result from the use of similar solutions for relatively short nozzles.

The method presented in Refs. 4 and 5 solves the non-similar laminar boundary-layer equations with or without first-order transverse curvature (referred to as second-order in Refs. 4 and 7), with or without velocity slip and temperature jump boundary conditions, and it has been shown to be accurate. However, the method has certain disadvantages: (1) the numerical integration scheme is that of Jaffe, Lind, and Smith (Ref. 7) which requires a relatively large amount of computer time, (2) the computer program is large and not necessarily simple to use, (3) the transformation variables are not amenable to internal flow problems, and (4) large flow expansions frequently require interpolating the solutions, changing the numerical step size across the nozzle, and resuming the calculations. These disadvantages discourage the use of the method described in Refs. 4 and 5.

The methods described in Refs. 3 through 5 assume that the nozzle flow consists of a viscous region and an inviscid (core) region. Some previous investigations which are not restricted to such flows, but permit viscous effects across the entire channel, are described in Refs. 8 through 10. These investigations are more suitable for the study of flow in microthrust rockets where the flow may be fully viscous. However, these methods also have certain disadvantages for the present application. The works of Adams (Ref. 8) and Williams (Ref. 9) are based on similar solutions, the conditions for which are not likely to be satisfied by a large class of practical problems. The work of Rae (Ref. 10) is a significant contribution to the study of low-density nozzle flows, and this method will be discussed further in this report. Numerical results from Rae's method will be compared with results from Ref. 5, results obtained in the present investigation, and experimental data. Rae's method was shown to give good results for fully viscous flows (Ref. 10); however, it will be shown herein to be less accurate than the method of Refs. 4 and 5 or the present method for the flow regime of interest in this investigation.

The objectives of the present investigation are: (1) to provide results for estimating the viscous effects in low-density converging-diverging nozzle flows based on certain flow parameters and nozzle geometries, and (2) to provide a fast, simple to use method for calculating viscous effects in low-density nozzle flows. To meet these objectives, the numerical integration scheme of Patankar and Spalding (Ref. 11) was used. In fact, the program used for this work was taken directly from Ref. 11 and then suitably modified for internal, low-density flows. The resulting program is similar to that of Mayne (Ref. 2) except for the numerical scheme used near the wall. Also, the

present work is based on a set of dimensionless equations which reduces the amount of input and provides more convenient solutions in the sense that they are applicable to flows which satisfy certain parameters rather than specific inputs of pressure, temperature, etc.

The following section describes the governing equations and boundary conditions, the variables used to nondimensionlize the equations, and the transformed equations. Section III describes briefly the numerical solution as well as some of the computational difficulties which have been encountered with this program. Section IV is devoted to numerical results and Section V to comparisons of the present results with previous theoretical investigations and experimental data. Some conclusions are given in Section VI.

SECTION II
BASIC EQUATIONS

The governing system of equations and the boundary conditions are presented in this section. Certain dimensionless and transformation variables are introduced and used to transform the governing equations for convenience of the numerical solution. Also, the boundary-layer displacement thickness is derived in the transformed plane. Although the program will solve the two-dimensional equations or axisymmetric equations with or without transverse curvature, only the equations appropriate to the latter with the transverse curvature terms retained are considered here in detail because they are more general. The two-dimensional equations can be obtained from the equations considered by setting $r \equiv 1$, and the axisymmetric equations without transverse curvature can be obtained by setting $r \equiv r_w$.

2.1 GOVERNING EQUATIONS

The governing system of equations is taken as that obtained by Probstein and Elliott (Ref. 12). The equations in curvilinear coordinates are:

Continuity Equation

$$\frac{\partial(\rho u r)}{\partial x} + \frac{\partial(\rho v r)}{\partial y} = 0 \tag{1}$$

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Momentum Equation

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{1}{r} \frac{\partial}{\partial y}\left(r\mu \frac{\partial u}{\partial y}\right)$$

(2)

Total Energy Equation

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{1}{r} \frac{\partial}{\partial y}\left\{ r\left[\frac{\mu}{Pr}\frac{\partial H}{\partial y} + \mu\left(1 - \frac{1}{Pr}\right)u\frac{\partial u}{\partial y}\right]\right\}$$

(3)

The total energy equation is obtained by multiplying the momentum equation by u and adding the result to the static energy equation. The coordinate system is defined in Fig. 1 (Appendix I) with the $r(x, y)$ term defined for internal flow as

$$r(x,y) = r_w(x) - y \cos \alpha$$

(4)

Probstein and Elliott (Ref. 12) obtained Eqs. (1) through (3) by an order of magnitude analysis of the general forms of the continuity equation, Navier-Stokes momentum equations, and energy equation. The assumptions made in the analysis were that the ratio of the boundary-layer thickness to the longitudinal radius of curvature of the body surface was small compared to unity, and the ratio of the boundary-layer thickness to the nozzle radius was on the order of unity. Therefore, Eqs. (1) through (3) are valid for nozzles which have a longitudinal radius of curvature much larger than the nozzle radius. This stipulation is normally satisfied by axisymmetric convergent-divergent nozzles used for low-density wind tunnels, rockets, and gas dynamic and chemical laser systems.

The axisymmetric boundary-layer equations without transverse curvature terms correspond to those which can be obtained from Eqs. (1) through (3) by replacing $r(x, y)$ with $r_w(x)$ as stated above. Because $r_w(x)$ is a function of x only, it can be eliminated from Eqs. (2) and (3), and therefore appears only in the continuity equation. The resulting set of equations can be used to describe internal or external boundary layers. It was shown in Ref. 5, by solutions with and without the transverse curvature terms, that the effect of transverse curvature is important for $\delta/r_w(x) \sim O(1)$.

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Implicit in Eq. (2) by the use of the total derivative of p with respect to x is the y component of the momentum equation as given by

$$-\frac{\partial p}{\partial y} = 0 \tag{5}$$

The validity of this equation is sometimes questioned for thick laminar boundary layers. However, the analysis of Probstein and Elliott (Ref. 12) indicates that this equation is consistent with the other equations in the governing set. Equation (5) was used in Refs. 4 and 5, and good agreement between calculated and measured boundary-layer profiles was obtained for flow conditions where 99 percent of the cross-sectional area of a nozzle was boundary layer.

For the boundary conditions at the edge of the boundary layer, it was assumed that an isentropic core flow exists along the nozzle centerline, from which the flow properties at the edge of the boundary layer can be calculated. The boundary conditions at the nozzle wall were taken as zero velocity and a prescribed wall temperature distribution or a prescribed wall heat-transfer distribution. Solutions were presented in Ref. 5 with and without velocity slip and temperature jump boundary conditions. For the conditions investigated in Ref. 5, it was found that the nozzle flow merged (that is, the boundary layer completely filled the nozzle) before velocity slip and temperature jump became significant. Therefore, no-slip wall boundary conditions were assumed adequate for this investigation.

The system of equations is completed by using the equation of state, $p = \rho RT$, and expressing the viscosity as some function of T, $\mu = \mu(T)$. The governing equations are next made dimensionless.

2.2 NONDIMENSIONALIZED EQUATIONS

Dimensionless variables are used to nondimensionalize the governing equations as follows:

$$\bar{x} = \frac{x}{r^*} \tag{6a}$$

$$\bar{\rho} = \frac{\rho}{\rho_o} \tag{6d}$$

$$\bar{p} = \frac{p}{\left(\frac{\gamma}{\gamma-1}\right)P_o} \tag{6g}$$

$$\bar{y} = \frac{y}{r^*} \tag{6b}$$

$$\overline{H} = \frac{H}{H_o} \tag{6e}$$

$$\bar{u} = \frac{u}{\left(H_o\right)^{\frac{1}{2}}} \tag{6h}$$

$$\bar{r} = \frac{r}{r^*} \tag{6c}$$

$$\bar{\mu} = \frac{\mu}{\mu_o} \tag{6f}$$

$$\bar{v} = \frac{v}{\left(H_o\right)^{\frac{1}{2}}} \tag{6i}$$

Using these equations in Eqs. (1) through (3) gives for the continuity, momentum, and total energy equation, respectively,

$$\frac{\partial(\bar{\rho}\,\bar{r}\,\bar{u})}{\partial \bar{x}} + \frac{\partial(\bar{\rho}\,\bar{r}\,\bar{v})}{\partial \bar{y}} = 0 \tag{7}$$

$$\bar{\rho}\,\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{\rho}\,\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{d\bar{p}}{d\bar{x}} - \frac{(2)^{\frac{1}{2}}}{Re_{o,r^*}}\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{y}}\left(\bar{r}\,\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{y}}\right) \tag{8}$$

and

$$\bar{\rho}\,\bar{u}\frac{\partial \bar{H}}{\partial \bar{x}} + \bar{\rho}\,\bar{v}\frac{\partial \bar{H}}{\partial \bar{y}} = \frac{(2)^{\frac{1}{2}}}{Re_{o,r^*}}\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{y}}\left\{\bar{r}\left[\frac{\bar{\mu}}{Pr}\frac{\partial \bar{H}}{\partial \bar{y}} + \bar{\mu}\left(1 - \frac{1}{Pr}\right)\bar{u}\frac{\partial \bar{u}}{\partial \bar{y}}\right]\right\} \tag{9}$$

where

$$Re_{o,r^*} = \frac{\rho_o(2H_o)^{\frac{1}{2}}r^*}{\mu_o} \tag{10}$$

A few comments are in order concerning the choice of the dimensionless variables. Note that $(H_o)^{1/2}$ was used to normalize the velocity components u and v instead of the more commonly used maximum velocity $(2H_o)^{1/2}$. The motivation for this was to recover the same form of the boundary-layer equations in dimensionless variables as in physical plane variables. If $(2H_o)^{1/2}$ were used in Eqs. (6h) and (6i) in place of $(H_o)^{1/2}$, then the term $\bar{\mu}[1 - (1/Pr)]\bar{u}\,\partial\bar{u}/\partial\bar{y}$ in the right hand side of Eq. (9) would be $2\bar{\mu}[1 - (1/Pr)]\bar{u}\,\partial\bar{u}/\partial\bar{y}$, which would not be the same as in Eq. (3). The maximum velocity $(2H_o)^{1/2}$ was used in the definition of Re_{o,r^*} for convenience. This introduces the term $(2)^{-1/2}$ as a coefficient of Re_{o,r^*}, but since Re_{o,r^*} is a constant for each solution, the constant $(2)^{-1/2}$ causes no inconveniences in the numerical solutions. Note that by defining a new viscosity, $\mu = (2)^{1/2}\bar{\mu}/Re_{o,r^*}$, Eqs. (7) through (9) are identical to Eqs. (1) through (3). This, in fact, was done in the numerical computations.

For a given set of initial conditions, boundary conditions, γ, Pr, nozzle geometry, etc., the solution to Eqs. (7) through (9) depends only on Re_{o,r^*} if a power law viscosity variation with temperature is assumed, i.e., $\bar{\mu} = \bar{T}^{\zeta} = (T/T_o)^{\zeta}$. However, if, for example, Sutherland's viscosity law is assumed, then the solutions will also depend on the absolute value of temperature. Because the calculation of

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viscosity is carried out in a separate subroutine in the computer program, it is convenient to use any viscosity law desired. Solutions will be presented for both Sutherland's law and power law variations of viscosity.

2.3 TRANSFORMED EQUATIONS

Equations (7) through (9) were transformed from the dimensionless physical $\bar{x}$ - $\bar{y}$ plane to the ξ - ω plane by the transformation variables

$$\xi(\bar{x}) = \bar{x} \tag{11}$$

and

$$\omega(\bar{x},\bar{y}) = \frac{\psi(\bar{x},\bar{y}) - \psi_I(\bar{x})}{\psi_E(\bar{x}) - \psi_I(\bar{x})} \tag{12}$$

where $\psi(\bar{x},\bar{y})$ is the stream function which identically satisfies the continuity equation, Eq. (7), i.e.,

$$\frac{\partial\psi}{\partial\bar{x}} = -\bar{\rho}\,\bar{v}\,\bar{r} \tag{13}$$

and

$$\frac{\partial\psi}{\partial\bar{y}} = \bar{\rho}\,\bar{u}\,\bar{r} \tag{14}$$

The transformation used is that due to von Mises with ω introduced to restrict the integration across the boundary layer from zero to unity. The subscripts I and E denote the inner and outer edge of the boundary layer, respectively, and ψ_I and ψ_E are functions of $\bar{x}$ only. The operators $(\partial/\partial\bar{x})$ and $(\partial/\partial\bar{y})$ are given by

$$\frac{\partial}{\partial\bar{x}} = \frac{\partial}{\partial\xi} + \frac{\partial\omega}{\partial\bar{x}}\frac{\partial}{\partial\omega} = \frac{\partial}{\partial\xi} + \left(\frac{\partial\omega}{\partial\psi}\frac{\partial\psi}{\partial\bar{x}} + \frac{\partial\omega}{\partial\psi_I}\frac{d\psi_I}{d\bar{x}} + \frac{\partial\omega}{\partial\psi_E}\frac{d\psi_E}{d\bar{x}}\right)\frac{\partial}{\partial\omega} \tag{15}$$

and

$$\frac{\partial}{\partial\bar{y}} = \frac{\partial\omega}{\partial\bar{y}}\frac{\partial}{\partial\omega} = \frac{\partial\omega}{\partial\psi}\frac{\partial\psi}{\partial\bar{y}}\frac{\partial}{\partial\omega} \tag{16}$$

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Using Eqs. (12) through (14) in Eqs. (15) and (16) and applying them to Eqs. (8) and (9) give for the momentum and total energy equations

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\bar{r}_I \dot{m}_I + \omega(\bar{r}_E \dot{m}_E - \bar{r}_I \dot{m}_I)}{\psi_E - \psi_I} \frac{\partial \bar{u}}{\partial \omega} = \frac{(2)^{\frac{1}{2}}}{Re_{o,r^*}} \frac{\partial}{\partial \omega} \left[\frac{\bar{r}^2 \bar{\rho} \bar{u} \bar{\mu}}{(\psi_E - \psi_I)^2} \frac{\partial \bar{u}}{\partial \omega} \right] - \frac{1}{\bar{\rho} \bar{u}} \frac{d\bar{p}}{d\bar{x}} \tag{17}$$

and

$$\frac{\partial \bar{H}}{\partial \bar{x}} + \frac{\bar{r}_I \dot{m}_I + \omega(\bar{r}_E \dot{m}_E - \bar{r}_I \dot{m}_I)}{\psi_E - \psi_I} \frac{\partial \bar{H}}{\partial \omega} = \frac{(2)^{\frac{1}{2}}}{Re_{o,r^*}} \left\{ \frac{\partial}{\partial \omega} \left[\frac{\bar{r}^2 \bar{\rho} \bar{u} \bar{\mu}}{(\psi_E - \psi_I)^2 \, Pr} \frac{\partial \bar{H}}{\partial \omega} \right] \right.$$
$$\left. + \frac{\partial}{\partial \omega} \left[\frac{\bar{r}^2 \bar{\rho} \bar{u} \bar{\mu}}{(\psi_E - \psi_I)^2} (1 - \frac{1}{Pr}) \frac{\partial(\bar{u}^2/2)}{\partial \omega} \right] \right\} \tag{18}$$

where

$$\frac{d\psi_I}{d\bar{x}} = -\bar{r}_I \dot{m}_I \tag{19}$$

and

$$\frac{d\psi_E}{d\bar{x}} = -\bar{r}_E \dot{m}_E \tag{20}$$

and where $\xi(\bar{x})$ has been replaced by $\bar{x}$. Except for the constant $(2)^{1/2}/Re_{o,r^*}$, Eqs. (17) and (18) are identical to the momentum and total energy equations for external flow which are solved numerically by the method of Patankar and Spalding (Ref. 11). The calculation of the mass transfer fluxes $\dot{m}_I$ and $\dot{m}_E$ will be discussed in Section III.

The boundary-layer displacement thickness was used to calculate an effective nozzle geometry which in turn was used to calculate an axial pressure distribution. The displacement thickness, δ^*, which takes into account transverse curvature is expressed by Probstein and Elliott (Ref. 12) as

$$\int_o^{\delta^*} 2\pi r \, \rho_E \, u_E \, dy = \int_o^{y_E} 2\pi r (\rho_E u_E - \rho u) dy \tag{21}$$

Solving Eq. (21) for δ^* gives

$$\frac{\delta^*}{r^*} = \frac{\bar{r}_w - \left[\bar{r}_w^2 - 2\cos\alpha\left(\bar{y}_E\bar{r}_w - \frac{\bar{y}_E^2\cos\alpha}{2} - \frac{\psi_E - \psi_I}{\bar{\rho}_E\bar{u}_E}\right)\right]^{\frac{1}{2}}}{\cos\alpha} \quad (22)$$

A quadratic equation for δ^* must be solved to obtain Eq. (22) from Eq. (21). The positive sign was chosen so that $\delta^*/r^* < \bar{r}_w/\cos\alpha$. The displacement thicknesses for axisymmetric flows without transverse curvature and planar flows can be obtained by using $(r_w)^\nu$ in place of r in Eq. (21). The displacement thicknesses are then given by

$$\frac{\delta^*}{r^*} = \bar{y}_E - \frac{(\psi_E - \psi_I)}{(\bar{r}_w)^\nu\bar{\rho}_E\bar{u}_E} \quad (23)$$

where $\nu = 0$ for planar flows and $\nu = 1$ for axisymmetric flows without transverse curvature.

SECTION III
NUMERICAL SOLUTION

The numerical solution of Eqs. (17) and (18) is discussed in this section. The basic scheme is briefly described and a listing of the computer code is presented. A description of the necessary inputs to the program is given, and some computational difficulties which have been encountered with the program are discussed.

3.1 BASIC SCHEME

As previously stated, the basic numerical integration scheme of Patankar and Spalding (Ref. 11) was suitably modified and used to solve the governing internal flow equations pertinent to the present investigation. Because the flow regime of the present investigation was entirely laminar, some subroutines of the computer code listed in Ref. 11 which were associated with turbulent flow were removed for the present code. Also, most, but not all, statements pertaining to turbulent flow were removed. The computer code as used for the present investigation is

listed in Appendix III. Essentially this same program was used on two different computers at AEDC, a Scientific Data Systems (SDS) 9300 and an International Business Machines (IBM) 370/155. The code in Appendix III is the one used on the IBM 370/155.

The basic numerical scheme used by Patankar and Spalding is discussed in detail in Ref. 11. One of the primary features of this finite difference technique is that the set of linear algebraic equations which must be solved has only three unknowns in each equation, and this set can be solved by simple successive substitution (Gauss reduction, or elimination, Ref. 13) rather than by matrix inversion. This technique provides a saving in computational time. For example, the time required to solve the algebraic equations by elimination (at a fixed $\bar{x}$ location) is proportional to the number of unknowns, whereas the time required to solve the equations by matrix inversion is proportional to the square of the number of unknowns (Ref. 13). On the IBM 370/155, a solution at a fixed $\bar{x}$ location required approximately 0.6 sec using 200 grid points across the boundary layer (ω direction) in single precision. The corresponding time required on the SDS 9300 was approximately 10 sec. The IBM 370/155 carried 8 digits in single precision, whereas the SDS 9300 carried 12.

The general method of solution consisted of matching the inviscid and viscous flow regions in the nozzle by iterating on the axial pressure distribution. An initial guess of the axial pressure distribution throughout the nozzle is made, and a solution is obtained. The displacement thickness calculated in this solution is used to obtain an effective nozzle geometry which in turn is used to calculate a new pressure distribution from one-dimensional, perfect gas, expansion theory. A typical iteration and convergence process is illustrated in Fig. 2. The use of one-dimensional perfect gas expansion theory as opposed to a method-of-characteristics solution, for example, seems justified on the basis of the agreement with experimental data in Section V. The same iteration process as used here and some suggestions for choosing the initial pressure distribution are discussed further by Whitfield and Lewis (Ref. 5).

The symbols and subroutines used in the Patankar and Spalding code are clearly defined and discussed in Ref. 11. Therefore, the remainder of this section is directed toward the modifications and additions which have been made to the code of Ref. 11.

3.2 INPUT CONDITIONS

The input requirements to this program are particularly simple. The input was modified somewhat from that of the original code (Ref. 11), and the input variables are described below in the order they are read-in in the present code (Appendix III).

SYMBOL	DESCRIPTION
KRAD	This input permits the treatment of plane flows and axisymmetric flows with first-order transverse curvature. Also, although not pointed out in Ref. 11, axisymmetric flows without transverse curvature can be treated by setting KRAD = 0. Plane flows can be treated by setting KRAD = 0 and $\bar{r}_w(\bar{x}) \equiv 1$ (or a constant). Axisymmetric flows including first-order transverse curvature are treated by setting KRAD = 1 (at least not zero) and using the actual geometry $\bar{r}_w(\bar{x})$.
IDIMEN	If the nozzle considered is two-dimensional, set IDIMEN = 0. This sets $\bar{r}_w(\bar{x}) \equiv 1$ in subroutine RAD. If the nozzle considered is axisymmetric set IDIMEN = 1.
NEQ	This is the number of partial differential equations to be solved. The code of Ref. 11 also includes the solution to the equation for the conservation of chemical species. However, this equation was never considered in this investigation and only the momentum and total energy equations were used, in which case NEQ = 2. The chemical species equation is, however, retained in the present code.
KEX	This input specifies the type of E boundary. It can be either 1, 2, or 3, according to whether the E boundary is a wall, free boundary, or a symmetry line, respectively. However, in the present investigation KEX was always 2, i.e., E was a free boundary, and certain modifications must be made to the present code if anything other than KEX = 2 is used.

KIN   This is similar to KEX except KIN specified the type of I boundary. For the present code KIN = 1 must be used; otherwise certain modifications must be made.

IHEAT   This is used in subroutine FBC and indicates a wall temperature boundary condition if IHEAT = 1 and a wall heat-transfer rate if IHEAT $\neq$ 1. In the present code, the only wall heat flux considered was zero, i.e., an adiabatic wall. If a heat flux other than zero is prescribed, then a few statements in subroutine SLIP must be modified (see Ref. 11). The only wall temperatute distribution considered in the present work was T_w/T_0 = constant. However, subroutine FBC is easily modified to accommodate any desired variation of T_w.

N   This is the number of strips across the boundary layer, i.e., in the ω direction. It must always be at least three less than the dimensions of the arrays of the variables across the layer, e.g., the maximum N which can be used with the dimensions of the arrays in the present code is 197.

REØRS   Reservoir Reynolds number, $Re_{0,r}{}^{*}$.

ZETA   Exponent in the power law variation of viscosity with temperature, $\mu \sim T^{\zeta}$

PR(1)   Prandtl number, Pr

GAM   Ratio of specific heats, γ

ALPHA   Nozzle wall half-angle, α

XR   Longitudinal radius of curvature of the nozzle upstream of the throat, $\overline{R}$

XL   Termination condition for the computations, maximum value of $\overline{x}$

USUP   This input controls the location of the E boundary. It is associated with the entrainment rate and it will be discussed in the following subsection. USUP was varied from 0.99 to 0.999, and was usually 0.995.

YSTART	Initial velocity and total enthalpy profiles were calculated from the expressions $u/u_E = 2\eta - \eta^2$ and $H/H_O = H_w/H_O + [1-(H_w/H_O)]u/u_E$ where $\eta = \bar{y}/\bar{y}_E$ and $\bar{y}_E$ = YSTART. Suggestions for choosing YSTART will be given in the following subsection. A typical value of YSTART is 0.5.
TWTØ	Ratio of wall to total temperature, T_w/T_O
XSTEP	Specification of the integration step-size in the $\bar{x}$ direction in terms of local wall radius, e.g., step size, DX, is given by the product of XSTEP times $\bar{r}_w$. A typical value of XSTEP is 0.05.
LMAX	Number of $\bar{x}$ locations where an input pressure is specified.
XX(L)	Array of $\bar{x}$ locations where an input pressure is specified. Array goes from 1 to LMAX.
PØP(L)	Input pressure for corresponding $\bar{x}$ location, XX(L). Array goes from 1 to LMAX.

The last three inputs, LMAX, XX(L), and POP(L), are read-in in subroutine PRE, the other inputs are read-in in subroutine BEGIN.

Because of the nature of the process involved in iterating on the axial pressure distribution, it is important to input a smooth initial pressure distribution. For the present computations, the initial pressure distribution was calculated using the actual geometry at and upstream of the throat and some effective inviscid nozzle wall downstream of the throat. (Actually, the slope of the assumed inviscid nozzle wall just downstream of the throat was matched to the nozzle wall half-angle to produce a smooth wall in order to have a smooth pressure distribution.) The simple program used to calculate the initial pressure distribution for the present computations is included in Appendix IV for convenience of users where such an approximation is adequate. The particular version of the code presented in Appendix IV uses $\delta^* \sim x^{3/2}$ for axisymmetric nozzles and $\delta^* \sim x$ for two-dimensional nozzles. Although $\delta^* \sim x^{3/2}$ was not used to calculate the initial pressure distribution for all axisymmetric nozzle computations presented herein, it appears to provide a reasonable approximation to the variation of δ^* in axisymmetric nozzles.

Inasmuch as several wind tunnel and laser nozzles have conical sections downstream of the throat and constant longitudinal radius of curvature for the upstream converging portions, this general nozzle geometry (which can be sufficiently described by the inputs ALPHA and XR) was considered in this investigation. However, subroutine RAD can be easily modified to include any geometry, such as for example contoured nozzles which were analyzed in Ref. 5 using the same governing equations as used here.

3.3 SOME POSSIBLE COMPUTATIONAL DIFFICULTIES

The mass-transfer rate, or entrainment rate, across the E boundary essentially governs the location of the edge of the boundary layer. This technique of locating the edge of the boundary layer has certain advantages in analyzing low-density nozzle flows; however, it might also cause some difficulties if not handled correctly. The entrainment rate was calculated in the present investigation by evaluating the momentum equation, Eq. (17), along a constant ω line, denoted as ω_B, near the E boundary. This technique is discussed in Ref. 11. The scheme requires the specification of the velocity along $\omega = \omega_B$ (where ω_B was taken as 0.9) at the next downstream station. This velocity is denoted as $\tilde{u}_B$ and it is suggested in Ref. 11 that it be taken as $\tilde{u}_B = 0.99 \, \overline{u}_E$, where $\overline{u}_E$ is the velocity at the edge of the boundary layer at the next downstream station. (Note that $\overline{u}_E$ can be calculated from the Euler equation since $d\overline{p}/d\overline{x}$ is presumed known for the particular iteration.) It was found in the present work, however, that more flexibility could be obtained with the program if $\tilde{u}_B$ was taken as $\tilde{u}_B = (USUP)\overline{u}_E$ and USUP was input for each solution. The quantity USUP provides a means of suppressing the outer edge of the boundary layer which is advantageous in treating flows which are nearly merged. For example, during the process of iterating on the axial pressure distribution, it was observed (Fig. 2, and also Ref. 5) that δ^* resulting from the first two iterations usually provided upper and lower bounds on the final converged δ^*. Therefore, if the flow is sufficiently rarefied, the pressure distribution resulting from the thinnest δ^* may be such that the following iteration would predict a merged flow when in fact the flow is not merged and could be calculated if a better guess for the initial pressure distribution could be made. In some cases the calculation of merged flow in the iteration process can be avoided by using a small value of USUP (a value of 0.99 is herein regarded as small and 0.999 is regarded as large) to suppress the edge of the boundary layer and prevent an indication of merging. The suppression seems to

apply only to the outer edge of the layer and the calculated profiles over most of the layer remain essentially unchanged. For this reason, the converged δ^*, or pressure distribution, obtained using a small USUP usually differs by a small amount from that obtained using a larger USUP, say 0.995. After convergence is obtained using a small value of USUP a final solution can be obtained using a larger value. The value of 0.99 for $\tilde{u}_B/\overline{u}_E$ as suggested in Ref. 11 seems to excessively suppress the boundary layer for the present internal flow calculations. For most solutions reported herein, USUP was 0.995.

The step-size along the $\overline{x}$ component was taken for most of the solutions as 5 or 7.5 percent of the local wall radius. This step-size was sufficiently small for most problems. However, if calculations of properties in the nozzle throat region are of particular interest, such as the nozzle wall heat-transfer rate, a smaller step-size may be desirable.

In some applications where nozzle exit properties are of particular interest, it may be desirable to conserve computer time and use a relatively large $\overline{x}$ component step-size. For such problems some difficulties might be encountered in starting the solutions. Consider a finite-difference form of Eqs. (19) and (20)

$$(\psi_E - \psi_I)_D = (\psi_E - \psi_I)_U + (\overline{r}_I \dot{m}_I - \overline{r}_E \dot{m}_E)_U (\overline{x}_D - \overline{x}_U) \qquad (24)$$

If Re_{o,r^*} and/or YSTART is small, then $(\psi_E - \psi_I)_U$ calculated from the initial profile will be small. Depending on the initial profiles and flow conditions, $\dot{m}_E$ may be positive for the first few stations and therefore $(\psi_E - \psi_I)_D$ could be less than $(\psi_E - \psi_I)_U$ for these first few stations and might even become negative. To circumvent this difficulty one could reduce the step-size $(\overline{x}_D - \overline{x}_U)$. However, if this is not desirable in view of computational time requirements, another approach is to increase YSTART in order to increase $(\psi_E - \psi_I)_U$ at the first station. The first few station solutions would consequently not be as accurate as solutions obtained by using a small $\overline{x}$ step-size. Therefore, although the solutions are started at the beginning of the converging portion of the nozzle upstream of the throat, this technique of increasing YSTART should be used with caution if accurate solutions (particularly properties which depend on derivatives near the wall such as heat-transfer rate) in the throat region are required. As stated previously, a typical value of YSTART was 0.5. The largest value of YSTART used to obtain solutions was unity, but this depends on XR.

Heat transfer and viscous effects change the effective nozzle throat from the location corresponding to the actual geometric nozzle throat. The subroutine NEWPPØ searches for the minimum effective nozzle radius and uses it for the throat in calculating a one-dimensional pressure distribution for the next iteration. However, if a sufficient number of solutions near the throat are not taken, then the true minimum area and its location used in calculating a new pressure distribution may not be accurately approximated. Then, the resulting $d\bar{p}/d\bar{x}$ in the throat region for the following iteration may not be smooth. In this case, the error in $d\bar{p}/d\bar{x}$ for each successive iteration would become worse. This is especially a problem in solutions for flows which have adiabatic or relatively hot nozzle walls. It is suggested that if the initial guess of the δ^* distribution is not a particularly good one, e.g., if the calculated δ^* at the exit is not within 20 to 30 percent of the assumed δ^*, then an improved pressure distribution should be calculated by the program in Appendix IV (or some similar method). This ensures a smooth pressure distribution, and since convergence is, in general, much faster in the throat region than further downstream where the relative displacement thicknesses are larger, few, if any, extra iterations are required.

Although areas have been pointed out where computational difficulties have been encountered with this program, it should also be pointed out that this numerical scheme is actually rather rugged, as for example compared to the method of Ref. 5. It is in general not sensitive to input conditions and seldom "blows up."

SECTION IV
NUMERICAL RESULTS

Solutions are presented in this section for typical nozzle configurations which provide some indication of the effect of Re_{o, r^*}, γ, Pr, ζ, T_w/T_o, α, transverse curvature, and two-dimensional versus axisymmetric flows. Table I (Appendix II) summarizes the conditions of the solutions presented in this section.

Of particular interest in nozzle flows is the displacement thickness, δ^*. By using δ^* and the nozzle geometry, an effective inviscid nozzle radius can be determined from which Mach number and other flow properties outside the boundary layer can be estimated. The displacement thicknesses for Conditions 1 to 8 of Table I are presented in Figs. 3 to 10.

Solutions were started at the beginning of the converging portion of the nozzles where $z/r^* = -3$. However, to conserve computing time, extremely small step-sizes were not utilized for all the solutions in this region (as discussed in Section III), and results are not presented in Figs. 3 to 10 for $z/r^* < -2$. Some solutions were repeated, however, with smaller step-sizes with no appreciable changes in the results presented.

At least for some values of $Re_{0,r}^*$ in Figs. 3 to 7 and 9 to 10, negative displacement thicknesses were calculated in the throat region. This is due to the relatively cool wall increasing the gas density and hence the local mass flux near the wall. The adiabatic wall results presented in Fig. 8 do not indicate negative δ^* for the same flow conditions. Similar results concerning the calculation of negative δ^* have been reported previously, e.g., Potter and Carden (Ref. 3) and Whitfield and Lewis (Refs. 4 and 5).

Some indication of the effect of using various gases in a fixed nozzle geometry with fixed flow conditions is provided in Figs. 3 to 5. The specific heat ratio, Prandtl number, and power law viscosity variation with temperature of Figs. 3 to 5 closely approximate the properties of carbon dioxide (CO_2), helium (He), and nitrogen (N_2), respectively. For example, by considering the nozzle exit displacement thickness for $Re_{0,r}^* = 10^4$, one observes that, for the indicated flow conditions, δ^* using He is about 200 percent of that when using CO_2, and δ^* using N_2 is about 150 percent of that when using CO_2.

Results are presented in Figs. 5 and 6 for identical conditions except for the nozzle wall half-angle. Figure 5 has a wall half-angle of 10-deg, and Fig. 6 has a half-angle of 15 deg. For the same geometric area ratio, the expansion process is more rapid for the $\alpha = 15$ deg nozzle than the $\alpha = 10$ deg. Also, the nozzle wall length for the same area ratio is longer for the 10-deg nozzle than for the 15-deg nozzle. The result is to produce a larger exit δ^* for the 10-deg nozzle than for the 15-deg nozzle for the same geometric area ratio. For $Re_{0,r}^* = 3 \times 10^3$, the 10-deg nozzle exit δ^* is about 18 percent larger than that of the 15-deg nozzle.

It should be pointed out that although δ^* is relatively small because of the cool walls, δ is not necessarily small. For example, in Fig. 5 for $Re_{0,r}^* = 10^3$, the flow merges (i.e., the boundary layer completely fills the nozzle) at a point where $\delta/r^* \approx 3$. For these conditions, $\delta^*/\delta \sim 0(1/10)$. The ratio δ^*/δ tends to increase with wall temperature.

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In general, the wall temperature seems to have a stronger effect on δ^* than on δ (Ref. 4). In Figs. 5, 7, and 8 the effect of nozzle wall temperature was investigated, with other conditions held constant, by considering $T_w/T_0 = 1/10$, $T_w/T_0 = 1/3$, and an adiabatic wall. For $Re_{0,r}{}^* = 10^4$, the ratio δ^*/δ at the nozzle exit was 0.30, 0.48, and 0.68, respectively.

The effect of Prandtl number was investigated by repeating the conditions of Fig. 5 but with $Pr = 1$ in place of 5/7. The results are presented in Fig. 9. The displacement thickness was found to increase for $Pr = 1$ by approximately 20 percent at the nozzle exit.

The conditions of Fig. 5 were also repeated using $\zeta = 1$ in place of $\zeta = 2/3$ to investigate the effect of the power law variation of vicosity with temperature. For $\zeta = 1$ the displacement thickness was found to decrease by 20 to 25 percent below that for $\zeta = 2/3$ (see Figs. 5 and 10). Note that, since $\bar{\mu} = \bar{T}^{\zeta}$ and $\bar{T} \leq 1$, then the flow is more viscous for $\zeta = 2/3$ than for $\zeta = 1$.

Velocity and temperature profiles calculated with and without transverse curvature terms are presented in Fig. 11 for Conditions 3 and 9 (Table I) with $Re_{0,r}{}^* = 3 \times 10^3$. Neglecting transverse curvature decreases the boundary-layer thickness for internal flow. The effect of transverse curvature is negligible for thin boundary layers, but may be significant for thick boundary layers. Further results and discussion concerning the effects of transverse curvature are reported in Ref. 5.

The displacement thicknesses of an axisymmetric and two-dimensional nozzle are presented in Fig. 12. For each nozzle, $A_e/A^* = 5$, and therefore, the two-dimensional nozzle is considerably longer since $\alpha = 10$ deg for each. The result is to produce a significantly larger displacement thickness for the two-dimensional nozzle than for the axisymmetric nozzle.

SECTION V
COMPARISONS WITH EXPERIMENTAL DATA AND
OTHER THEORETICAL RESULTS

Curves of $Re_{0,r}{}^*/(p_0 r^*)$ versus total temperature for various gases are presented in Fig. 13 for the convenience of determining $Re_{0,r}{}^*$. The viscosities used for all gases in Fig. 13 except carbon dioxide (CO_2) are

those given by Svehla (Ref. 14). The viscosity of CO_2 was taken from Table 8.4-2 of Hirschfelder, Curtiss, and Bird (Ref. 15).

Highly viscous nozzle flows are usually associated with nozzles of small physical size. The spatial resolution of experimental measurements in such nozzles is obviously restricted. However, highly viscous flows can also be produced in large nozzles if sufficiently large volume flows can be pumped at low pressures, thereby permitting more detailed investigations of the boundary layer. Such investigations are possible in the Aerospace Research Chamber 10V in the VKF at AEDC. The present and some previous calculation methods will be compared with experimental data taken in Chamber 10V using a nominal Mach three nozzle, denoted M3 nozzle. This is a 10-deg half-angle conical nozzle with $d^* = 27$ cm, $d_e = 76.2$ cm, and $R = 11.2$ cm. The walls of this nozzle were cooled with liquid nitrogen to maintain a constant nozzle wall temperature of about 85°K.

Calculated and measured pitot pressure profiles at the nozzle throat are presented in Fig. 14. Nitrogen was the test gas used for these and all other experimental data presented in this report. The present results are in good agreement with the results using the method of Ref. 5. The calculated profiles are in relatively good agreement with the measured profile for these conditions.

Calculated and measured pitot pressure profiles at the nozzle exit are presented in Fig. 15. This calculation was performed using $\zeta = 2/3$, whereas some other results presented herein, e.g., Fig. 14, were obtained using Sutherland's viscosity law for nitrogen. However, over the temperature range of the experimental data taken in Chamber 10V, there was negligible difference in the results using either viscosity variation with temperature.

Present calculations are compared to calculations by the method of Rae (Ref. 10) and experimental data in Figs. 16 and 17. The present results are in relatively good agreement with the experimental data, but Rae's method is found to overestimate the size of the viscous region for these conditions. Results were also presented in Ref. 5 for the conditions of Fig. 17. The calculated profile from Ref. 5 is almost identical with the present result in Fig. 17 (see Ref. 5) and is not presented.

Pitot pressure profiles at about 7 throat radii downstream of the M3 nozzle throat are presented in Fig. 18 for $Re_{o,r^*} = 1170$. A relative minimum exists at the nozzle centerline in the experimentally

measured profile, and slight "humps" or relative maximums exist near the edge of the boundary layer. It should be pointed out that the existence of humps does not necessarily imply that the flow is non-isentropic. Imagine a radial pitot pressure profile in an inviscid contoured supersonic nozzle which is sufficiently far downstream of the throat to pass through a portion of the uniform parallel flow near the centerline. Such a pitot profile would have lower values near the nozzle centerline than near the wall because of the larger Mach numbers near the centerline. However, the matching of such an inviscid flow with a realistic viscous flow requires the pitot pressure near the wall to approach the local static pressure which is less than the centerline pitot pressure. Therefore, it is not difficult to conceive of humps in such a radial pitot profile because of the matching of inviscid and viscous flow. This argument is based on flow in a contoured nozzle. It is applied to the present case because the displacement thickness effectively contours the nozzle. A more accurate approach of investigating such flows would be to remove the present assumption of one-dimensional inviscid flow outside the boundary layer and obtain more accurate solutions, such as, inviscid method-of-characteristics solutions. However, the pitot pressure profile is relatively well predicted in Fig. 18, and for this work, the assumption of one-dimensional inviscid flow outside the boundary layer is considered acceptable.

Just as radial pitot profiles with humps do not necessarily imply that the flow is non-isentropic, a flat radial profile does not necessarily imply that the flow is isentropic. Consider the pitot pressure data in Fig. 19. Both the calculated and measured profiles are relatively flat for y/r_w larger than about 0.7. However, in this case the pitot profile is not a good measure of the extent of the nozzle wall viscous effects as shown in Fig. 20. From Fig. 20, the boundary-layer thickness as estimated from the pitot profile is about 55 percent of the nozzle radius, whereas it is calculated to be actually over 80 percent. The pitot profile for the case in Fig. 20 implies that 20 percent of the cross-sectional area of the nozzle at this point is core flow, whereas actually less than 4 percent is core flow. The reason a pitot profile might lead one astray is associated with the temperature or thermal boundary layer since pitot pressure depends, among other things, on $u/(T)^{1/2}$. The velocity variation is usually well behaved and fairly accurately predicted by simple analytical expressions based on boundary-layer thickness (Ref. 4); however, this is not the case with the thermal boundary layer. The temperature variation depends not only on local wall and edge values and gradients but also on the upstream conditions.

Therefore, some consideration should be given to the temperature variation in order to place limits-of-confidence on the use of pitot pressure as an indication of the nozzle wall viscous effects.

Carden (Ref. 16) measured local heat-transfer coefficients in an axisymmetric nozzle for $Re_{o,r^*} = 5 \times 10^3$. The experimental data were compared with calculated heat-transfer coefficients in Refs. 5 and 16. However, Whitfield made a mistake in Ref. 5 and presented solutions for $r^* = 0.262$ cm instead of $d^* = 0.262$ cm (0.103 in.) which corresponds to the actual nozzle throat dimension. Because of this error, the calculated heat-transfer coefficients (Ref. 5) from the iterated solutions were significantly below the experimental data of Carden (Ref. 16). Heat-transfer coefficients were calculated using the present method but with the inappropriate throat dimension of $r^* = 0.262$ cm, and good agreement was obtained with the calculated results of Ref. 5. Results were also obtained with the present method using the proper throat dimension of $d^* = 0.262$ cm, and good agreement with the experimental data was obtained as shown in Fig. 21. Also, in Fig. 21 are results from two of the methods Carden (Ref. 16) used for calculating the heat-transfer coefficient and one solution from Ref. 5. Although the result from Ref. 5 which is presented in Fig. 21 was not iterated to include the higher-order displacement effect, the result was obtained using the pressure distribution corresponding to the proper nozzle geometry and, therefore, is included in Fig. 21. This result from Ref. 5 is in relatively good agreement with the present result. All calculation methods underestimate the most upstream uncorrected experimental data point in Fig. 21. However, Carden points out that radiation from the arc used to heat the gas could increase the total heat-transfer rate to this portion of the nozzle. No corrections for radiation heat transfer were made to the experimental data.

It might be pointed out that the heat-transfer rate printed out in the present program, denoted as AJI(1), is equal to $q_w/(\rho_o H_o^{3/2})$. Although made dimensionless, the numerical scheme used in Ref. 11 to calculate AJI(1) is retained in the present code in subroutine WALL.

SECTION VI
CONCLUSIONS

The present results were shown to be in good agreement with results from the method of Ref. 5. Although it was noted during the present investigation that more axial stations were required using the

present program than that of Ref. 5 to obtain the same degree of accuracy (presumably this is due to the transformation variables used in Ref. 5, which are advantageous for thin boundary layers but disadvantageous for thick internal boundary layers), the present program is much smaller, faster and easier to use. Comparisons with results from Rae's method (Ref. 10) and experimental data indicated the present method to be more accurate than Rae's for the conditions considered. It should be remembered, however, that the present work is not applicable to merged flows without modification, whereas Rae's method is more applicable to merged flows.

Consistent agreement was obtained between the present results and experimental data. Except for the heat-transfer data, the experimental data were taken in a relatively large nozzle where the boundary layer could be accurately probed. Similar agreement was obtained in Ref. 2 where extensive data were obtained in small nozzles, $r^* \sim 0$ (0.1 cm). The numerical method used in Ref. 2 was developed by Mayne (Ref. 2), as stated previously, and differs from the original Patankar and Spalding method (Ref. 11) primarily in the numerical scheme near the wall.

The present method has not been modified for use in nozzle design; however, it is a straightforward matter to make such a modification (see Ref. 5). To design a nozzle, a new nozzle geometry, $r_w(x)$, is calculated from the displacement thickness from each iteration, and the new geometry is used for the next iteration while the desired axial pressure distribution is maintained for each iteration. It was found in Ref. 5 that convergence was, in general, more rapid in iterating on the nozzle geometry than on the axial pressure distribution.

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APPENDIXES
I. ILLUSTRATIONS
II. TABLE
III. BOUNDARY-LAYER COMPUTER CODE
IV. INITIAL PRESSURE DISTRIBUTION
COMPUTER CODE
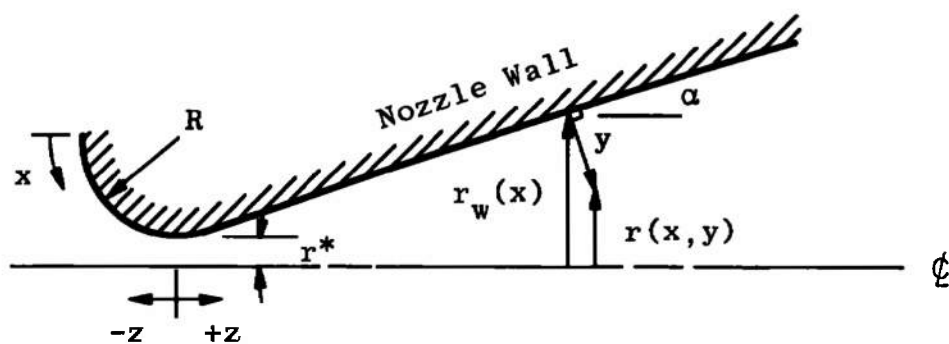
Fig. 1 Definition of Coordinate System
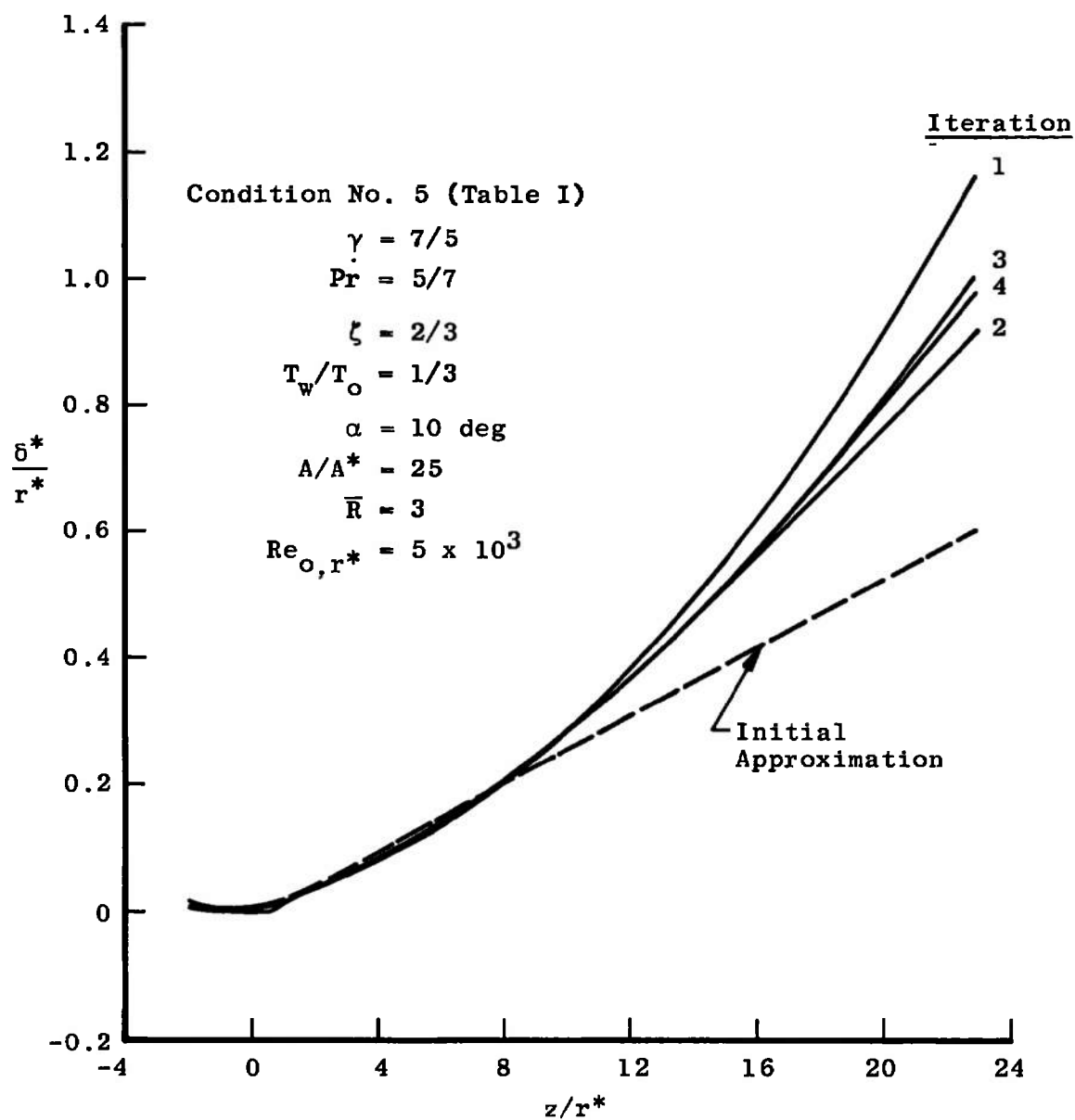
Fig. 2 Displacement Thickness of Successive Iterations
for Condition No. 5 (Table I)
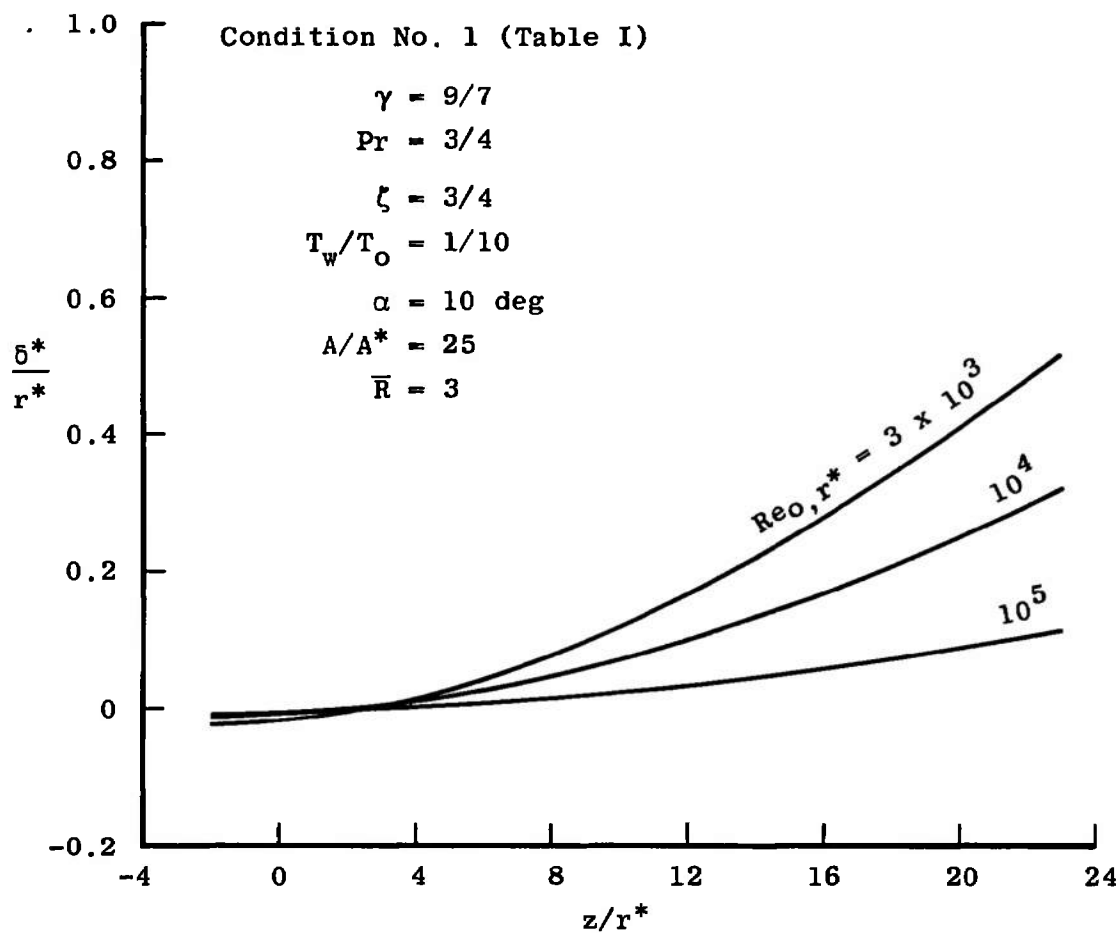
Fig. 3 Displacement Thickness for Condition No 1 (Table I)
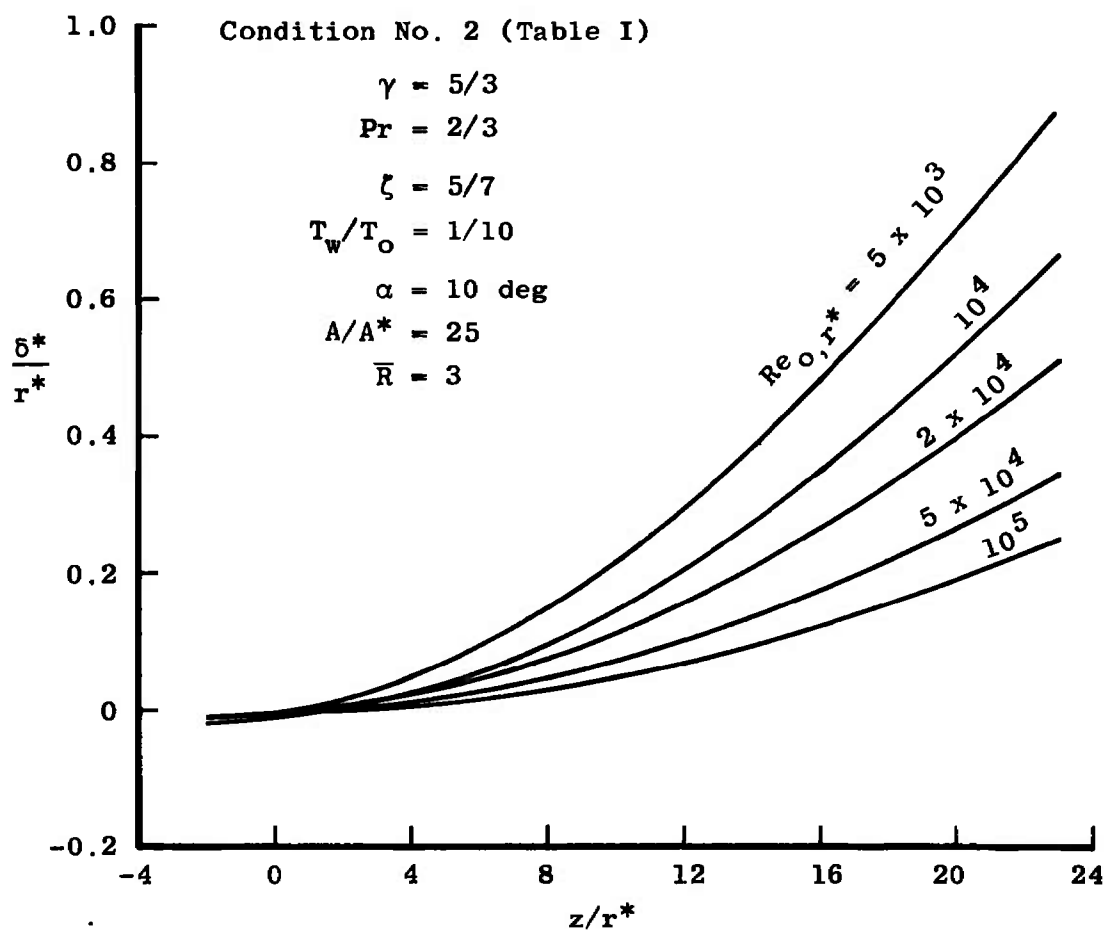
Fig. 4 Displacement Thickness for Condition No. 2 (Table I)
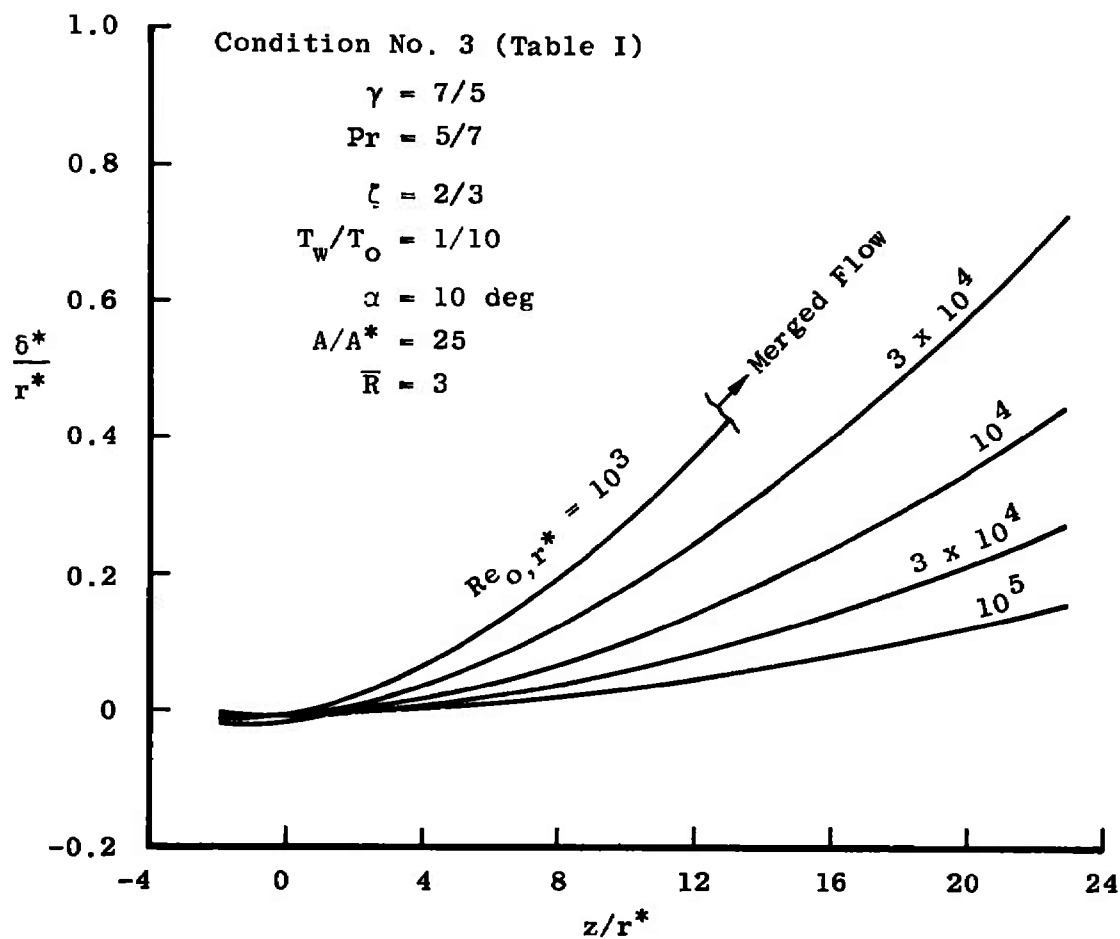
Fig. 5 Displacement Thickness for Condition No. 3 (Table I)
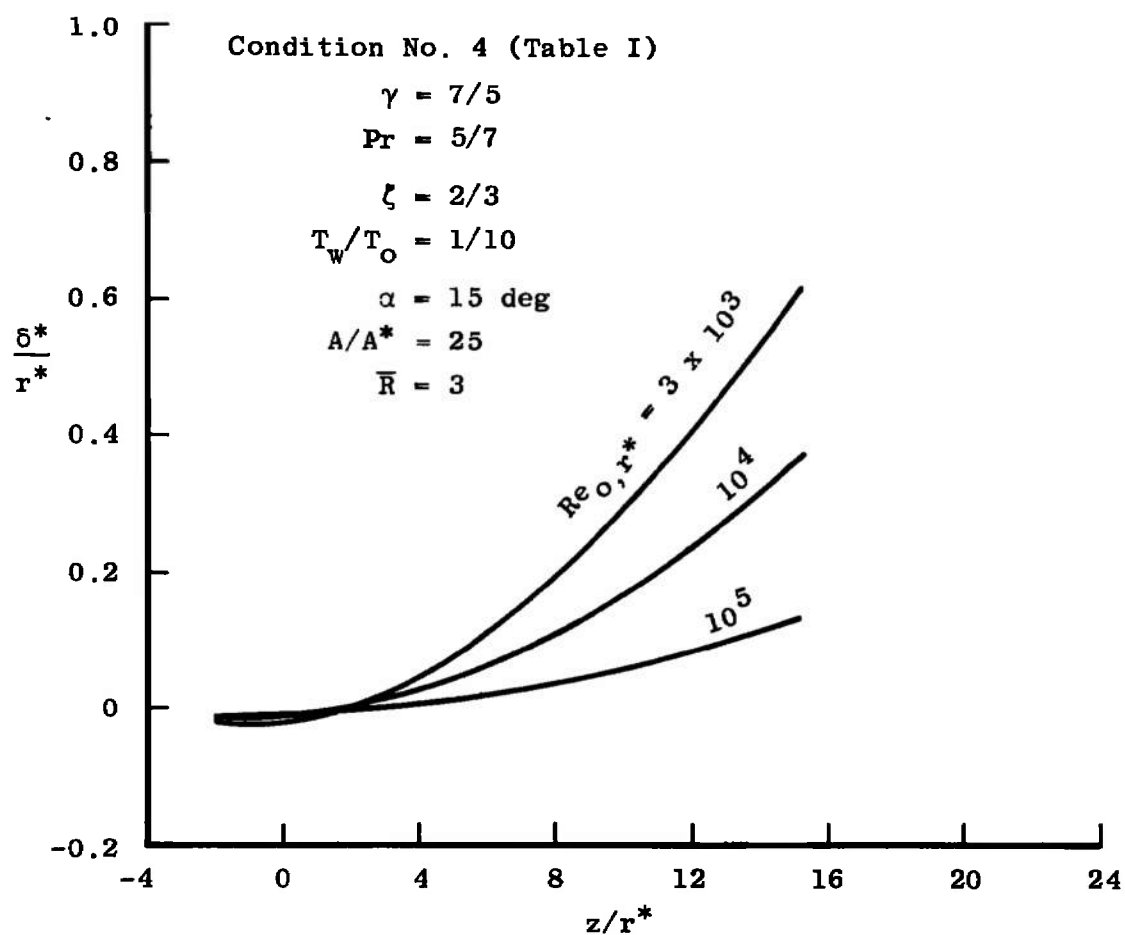
Fig. 6 Displacement Thickness for Condition No. 4 (Table I)
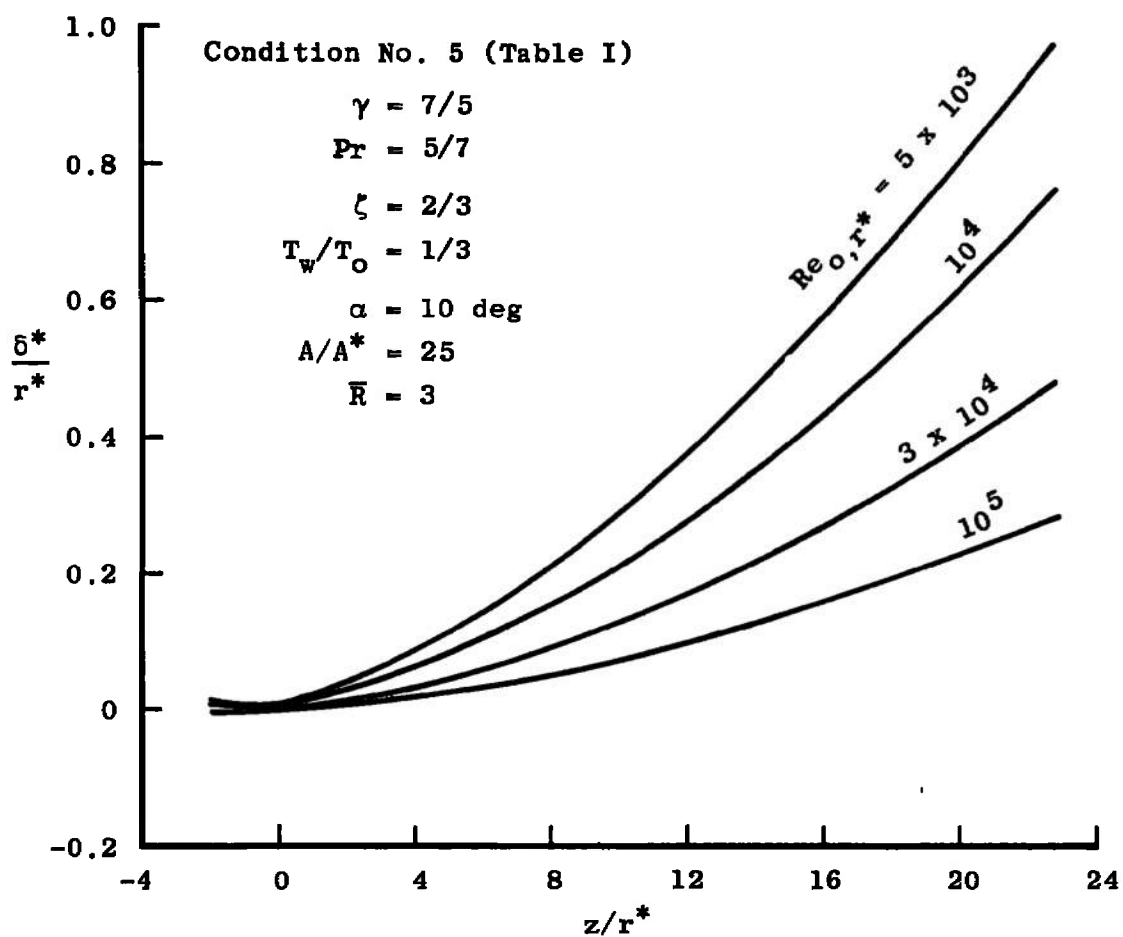
Fig. 7 Displacement Thickness for Condition No. 5 (Table I)
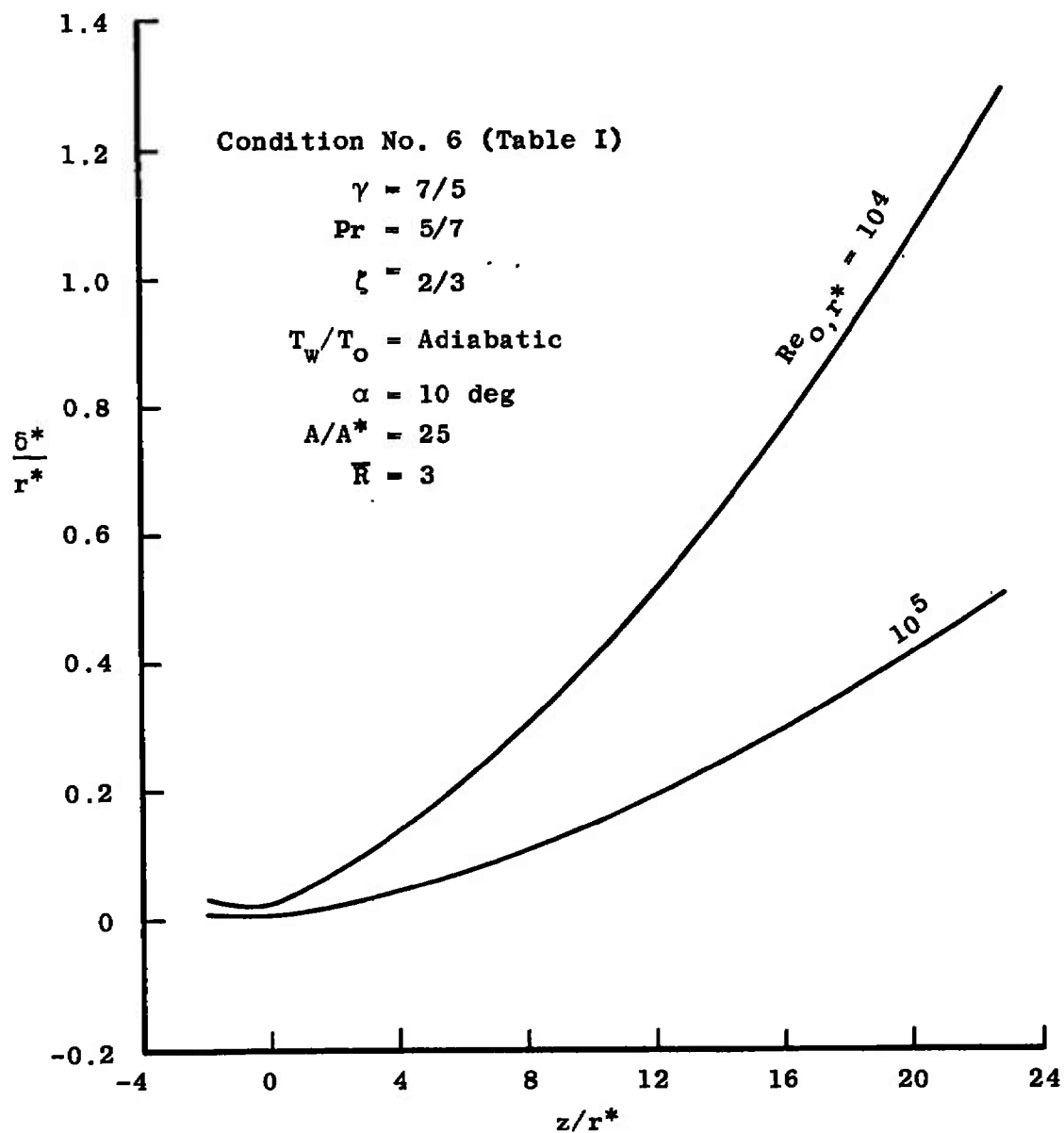
Fig. 8 Displacement Thickness for Condition No. 6 (Table I)
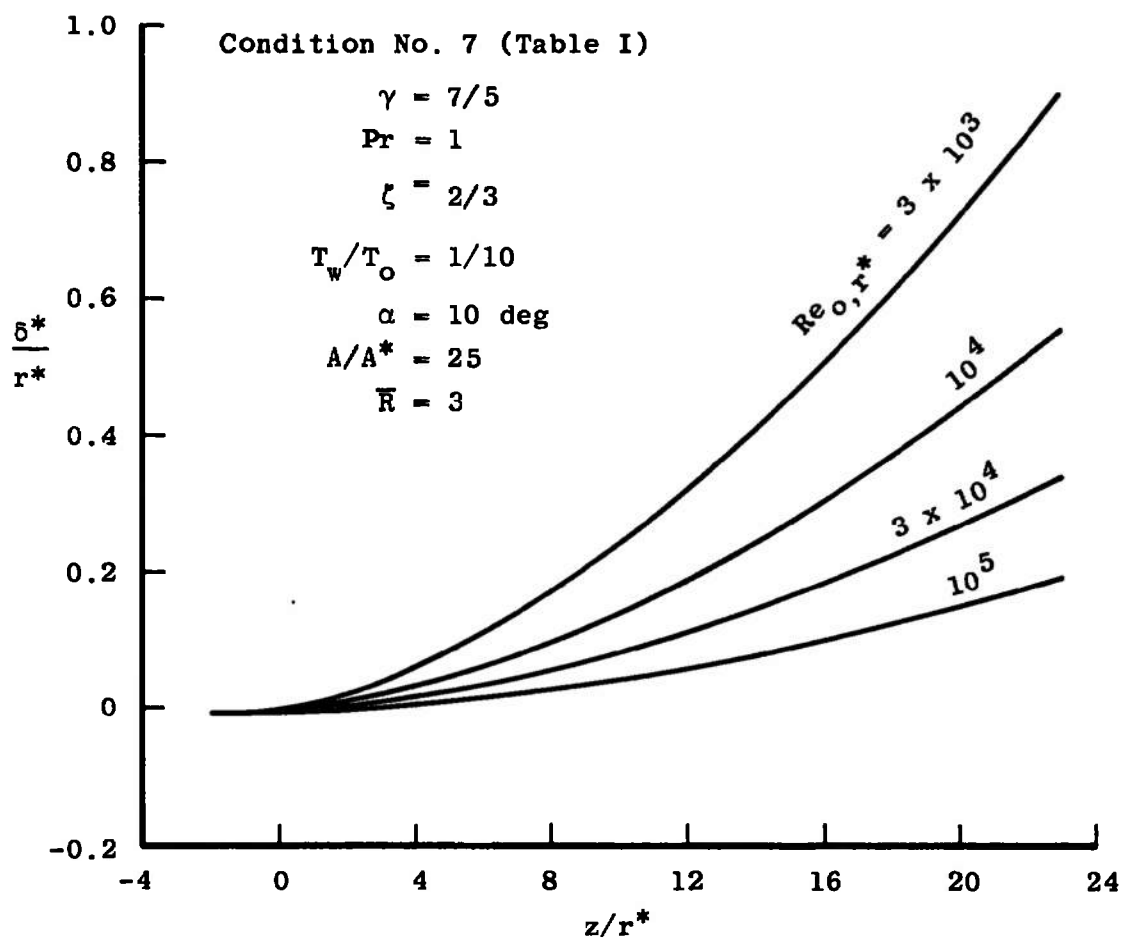
Fig. 9 Displacement Thickness for Condition No. 7 (Table I)
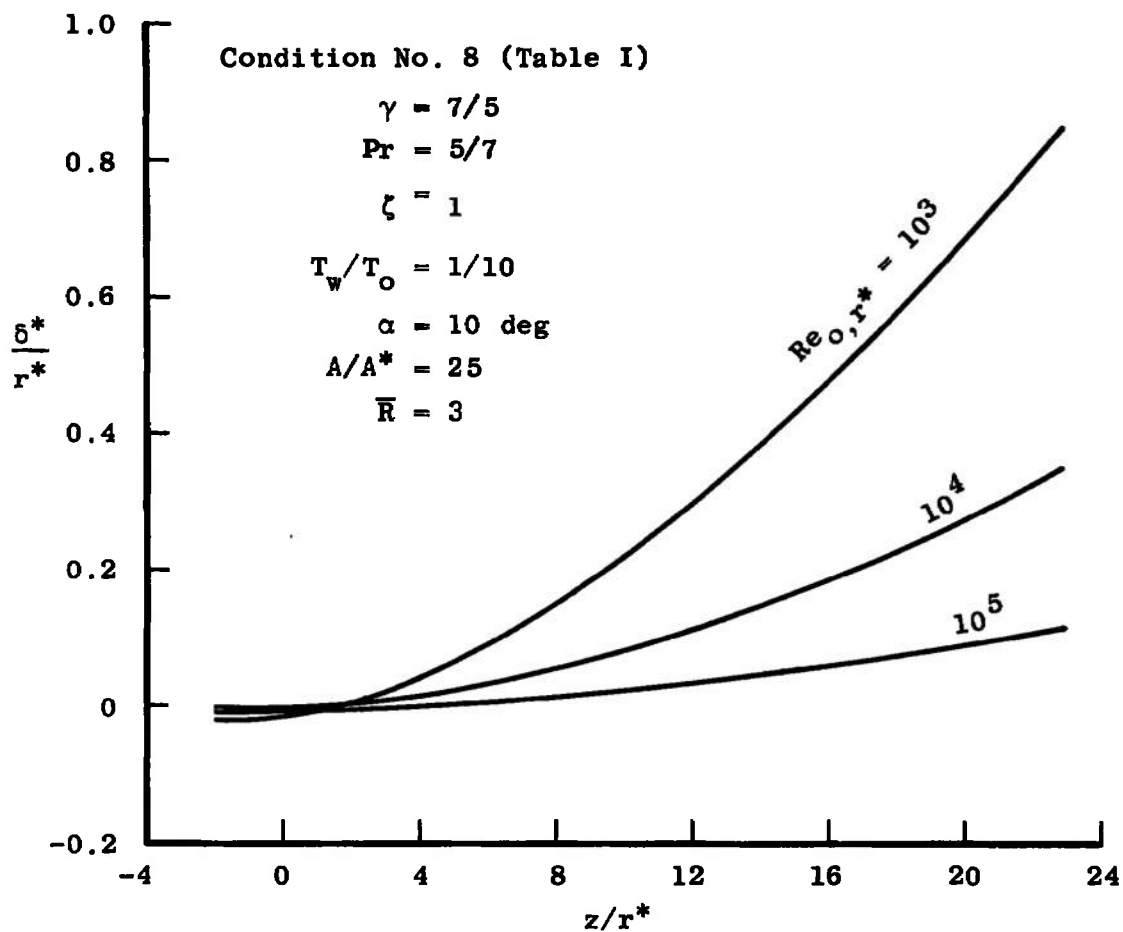
Fig. 10 Displacement Thickness for Condition No. 8 (Table I)
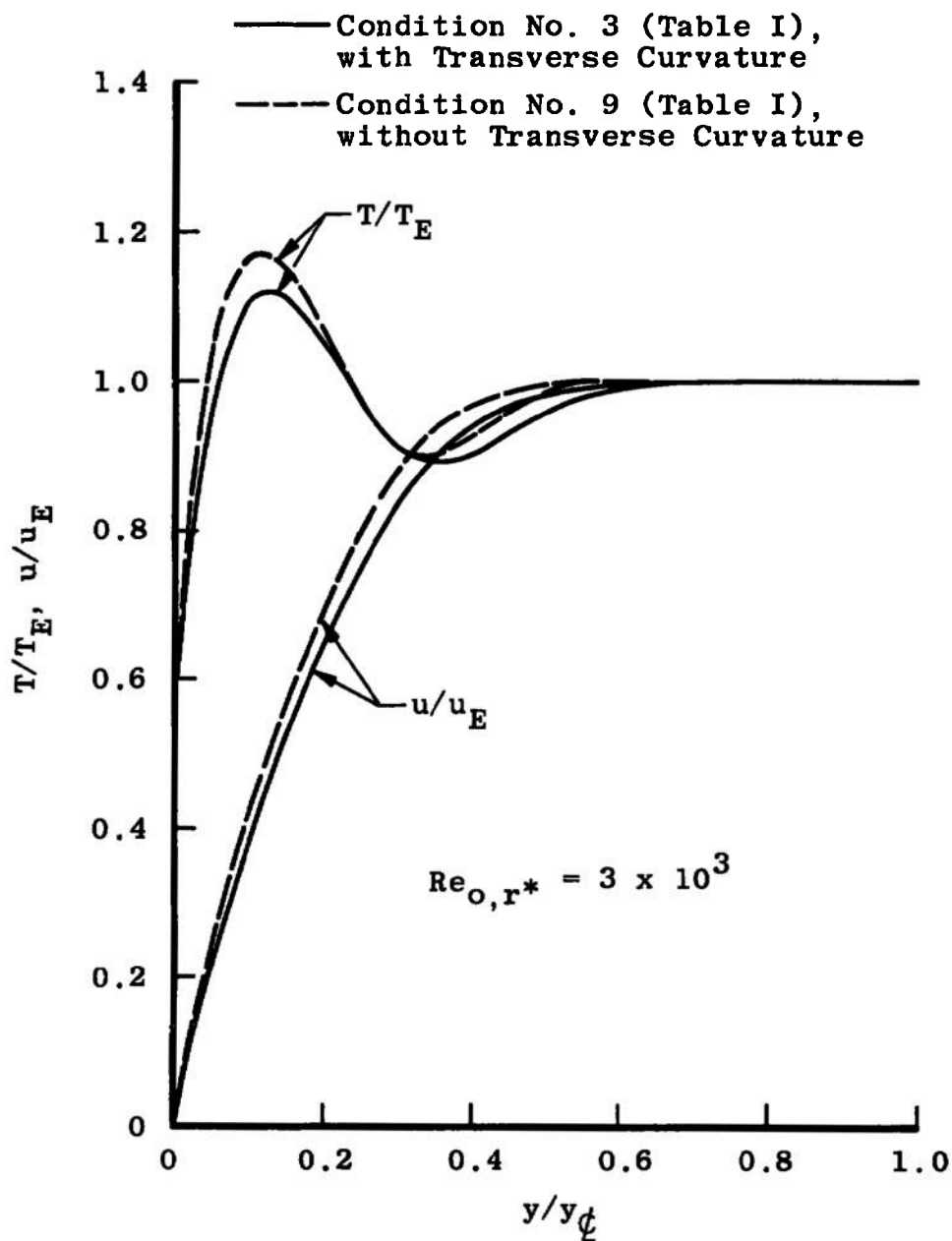
Fig. 11 Nozzle Exit Velocity and Temperature Profiles
for Condition Nos. 3 and 9 (Table I)
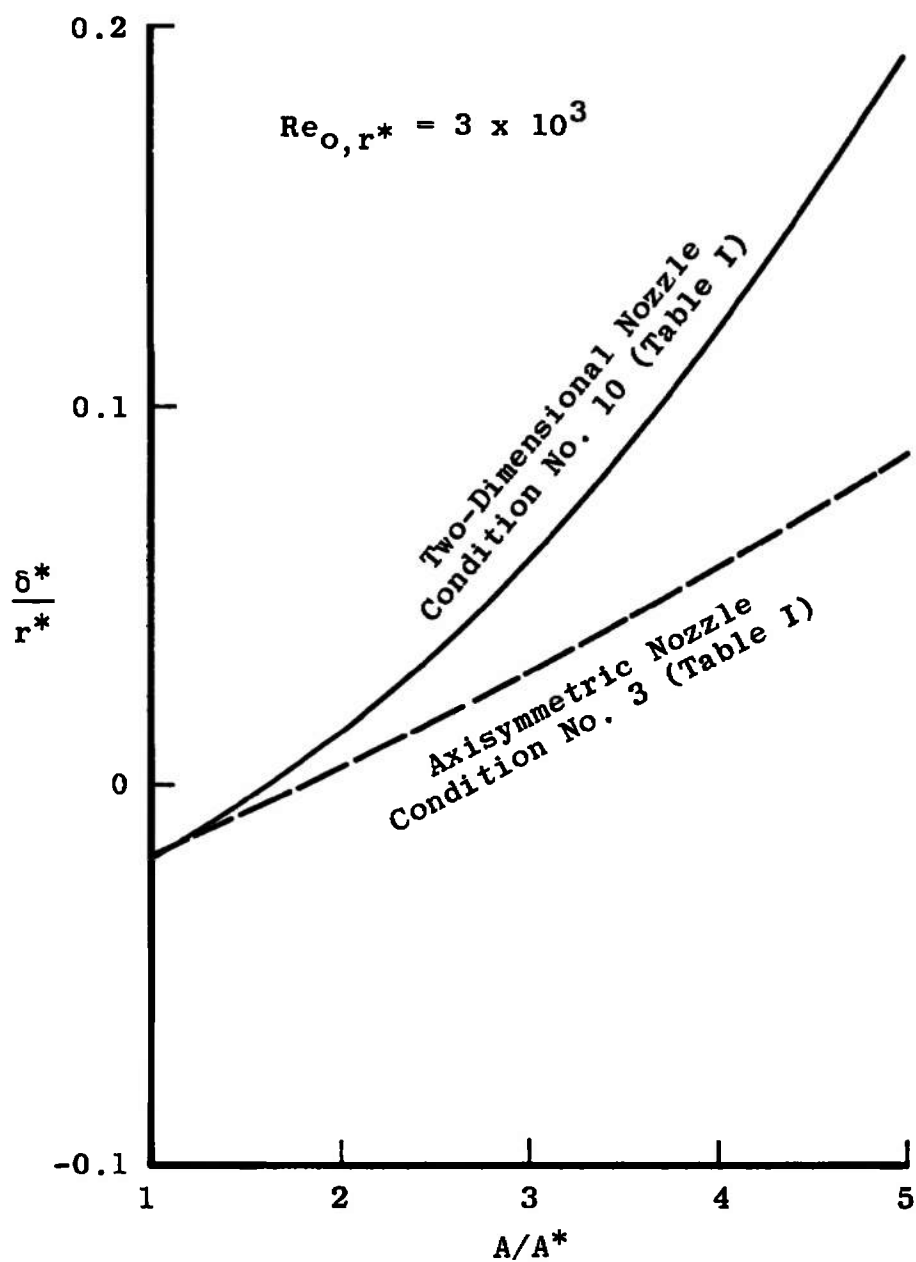
Fig. 12 Displacement Thicknesses of an Axisymmetric and
Two-Dimensional Nozzle with Equal Area Ratios
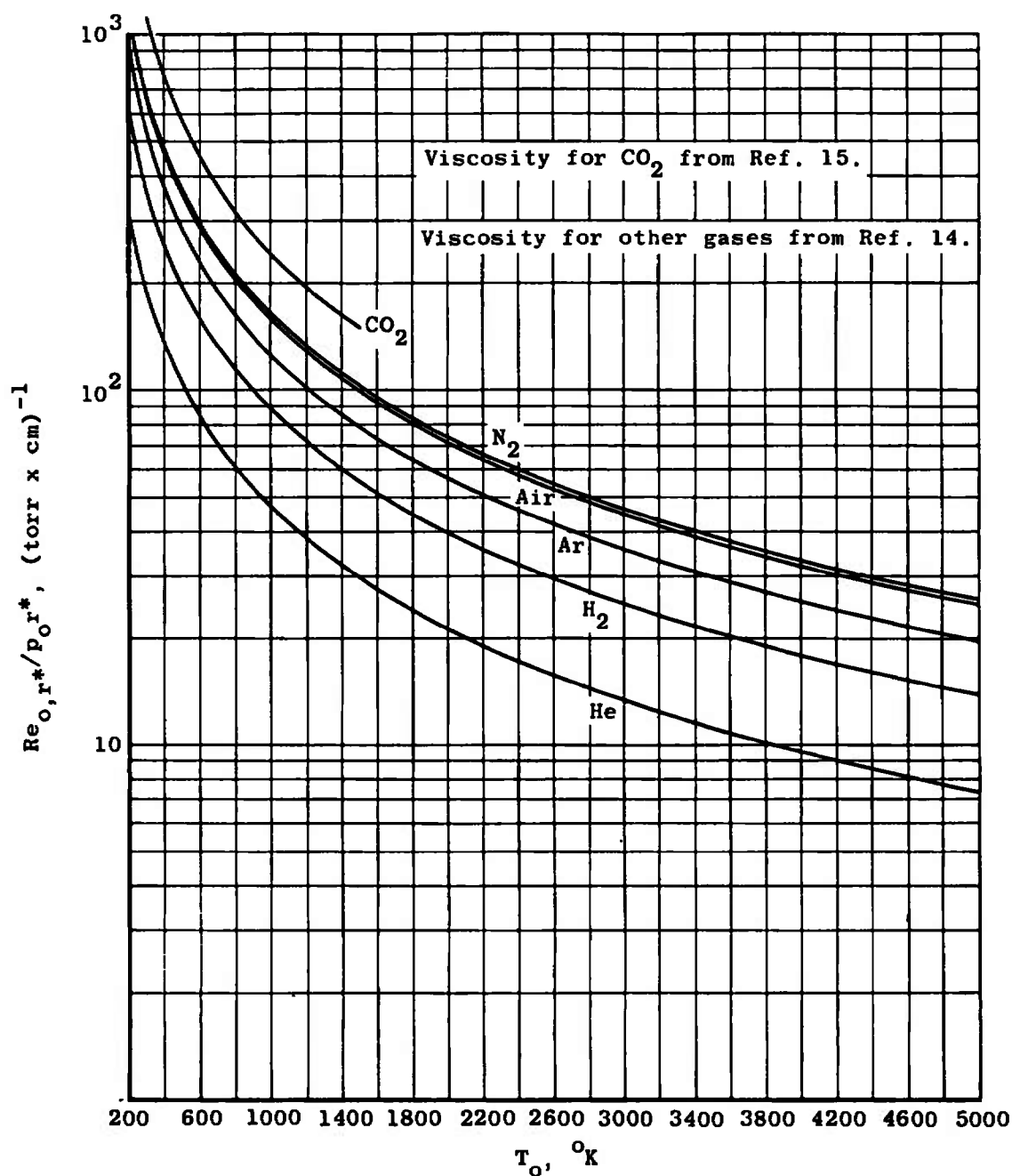
Fig. 13 Reservoir Reynolds Number as a Function
of p_o, T_o, and r^* for Various Gases
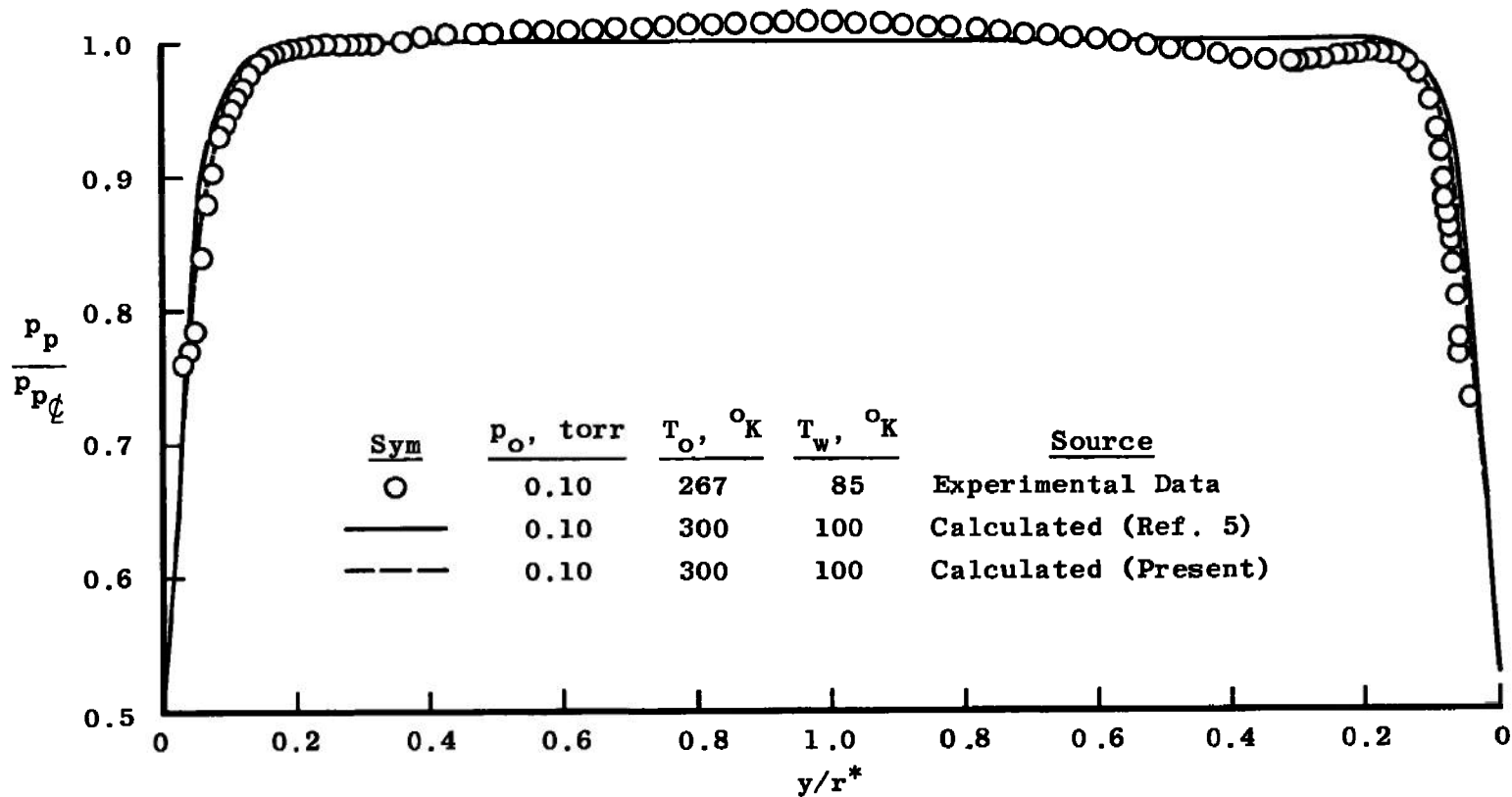
Fig. 14 Calculated and Measured Pitot Pressure Profiles
at the M3 Nozzle Throat for $Re_{o,r*}$ = 900
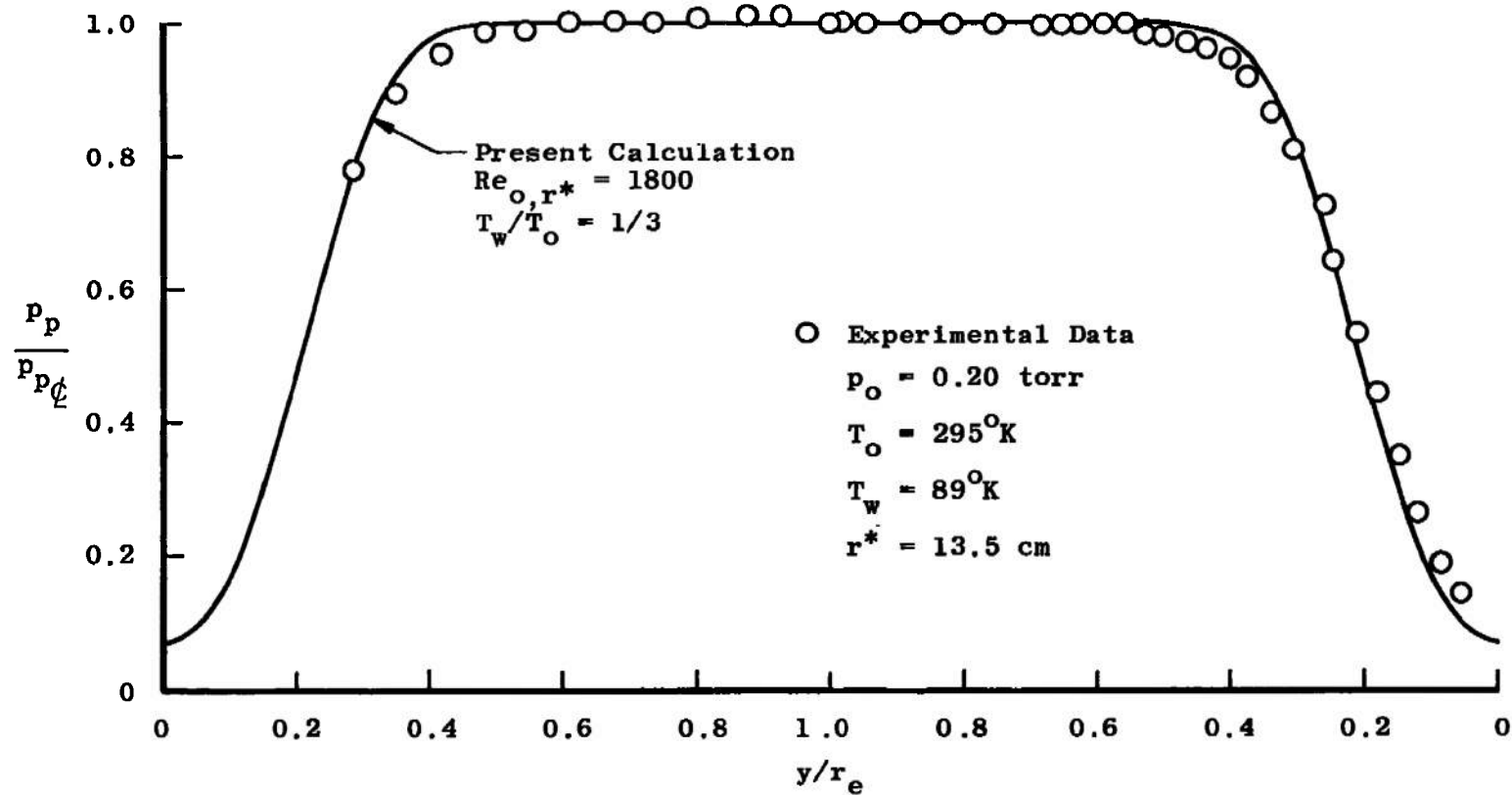
Fig. 15 Calculated and Measured Pitot Pressure Profiles
at the M3 Nozzle Exit for $Re_{o,r*} = 1800$
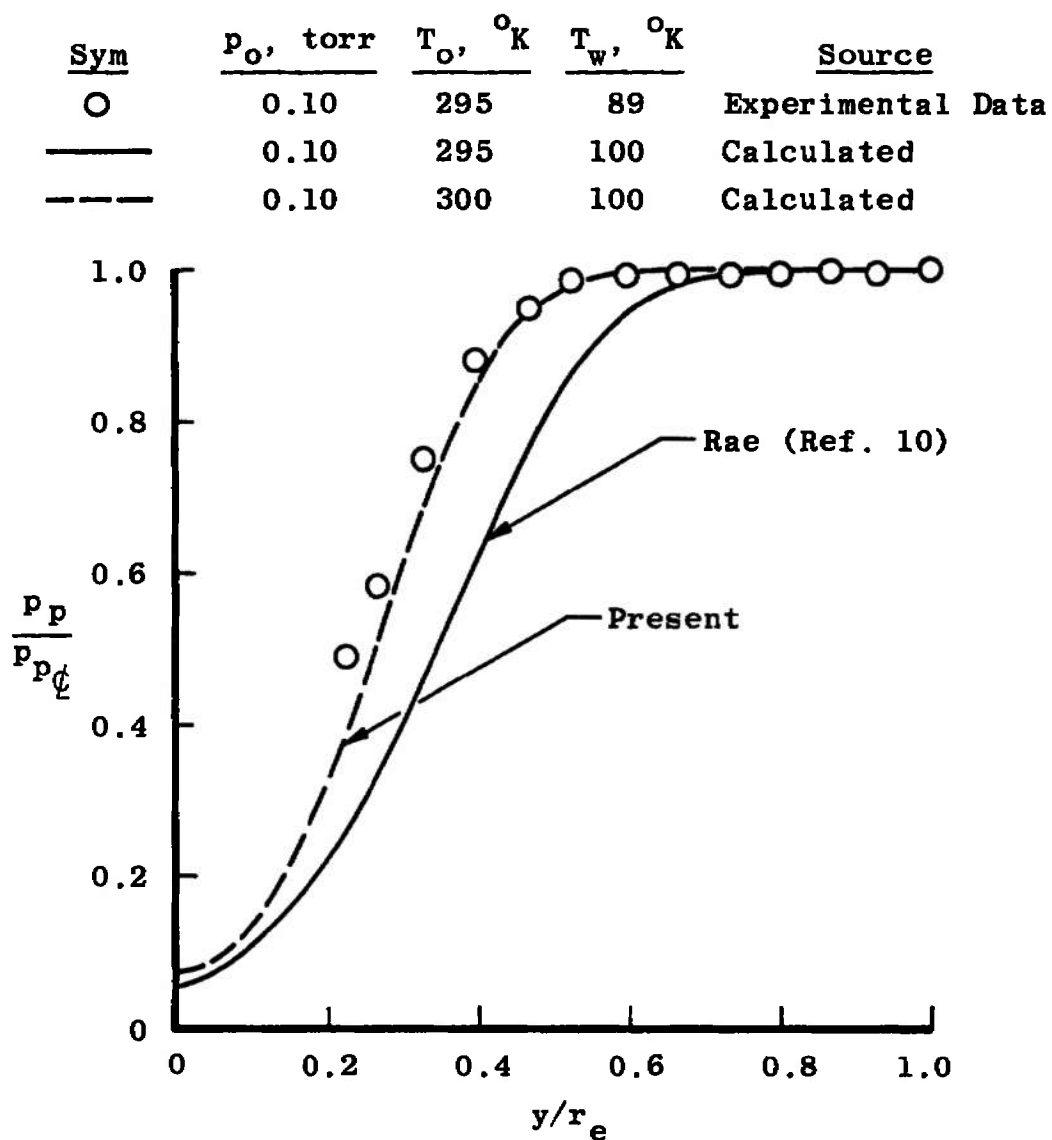
Sym	p_o, torr	T_o, $^{\circ}K$	T_w, $^{\circ}K$	Source
O	0.10	295	89	Experimental Data
——	0.10	295	100	Calculated
— — —	0.10	300	100	Calculated

Fig. 16 Calculated and Measured Pitot Pressure Profiles
at the M3 Nozzle Exit for $Re_{o,r\bullet}$ = 900

Sym	p_o, torr	T_o, ^{o}K	T_w, ^{o}K	Source
O	0.50	280	89	Experimental Data
——	0.50	300	100	Calculated
— —	0.50	300	89	Calculated
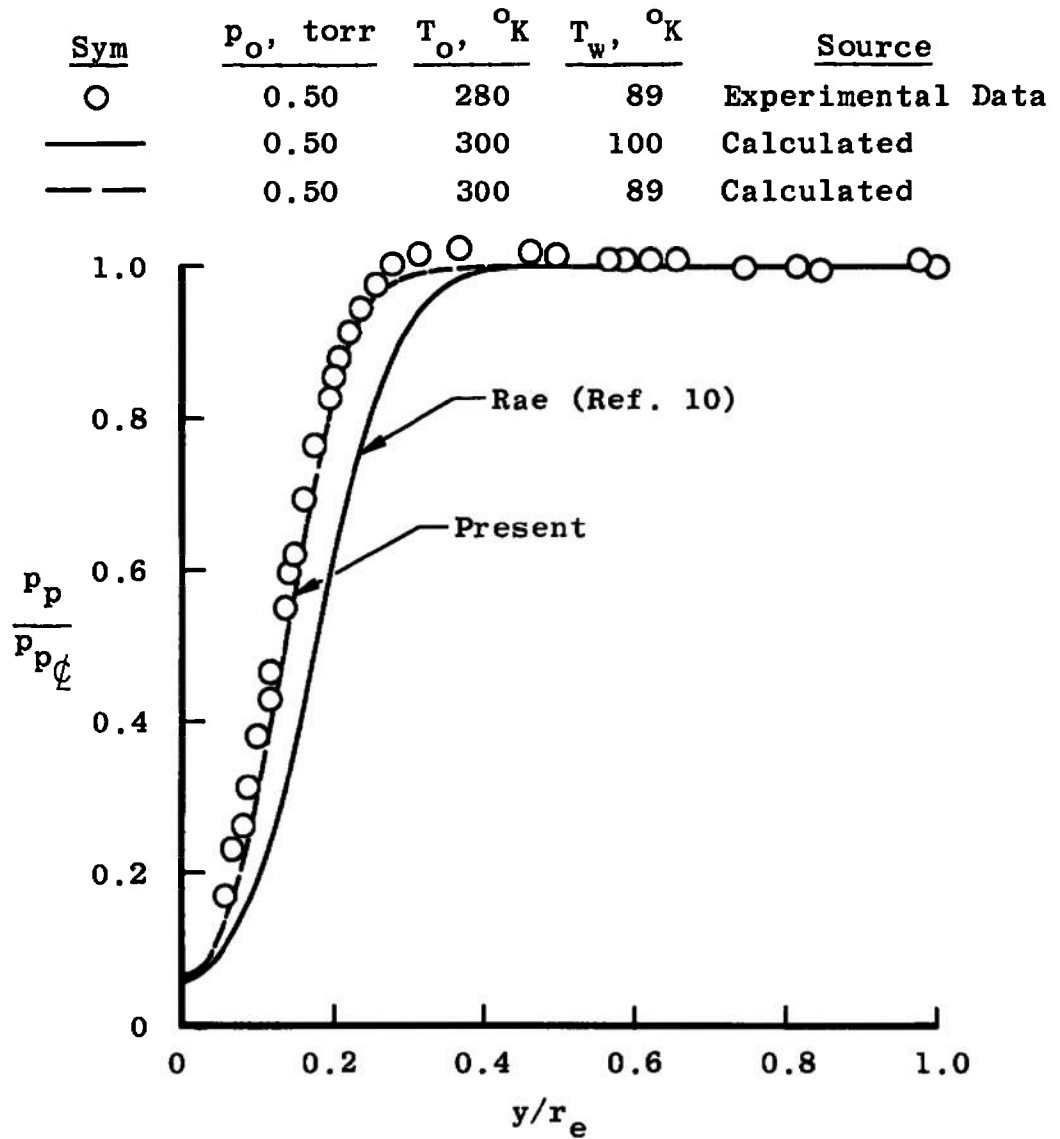
Fig. 17 Calculated and Measured Pitot Pressure Profiles
at the M3 Nozzle Exit for $Re_{o,r*}$ = 4500

$\mathrm{Re}_{o,r*} = 1170$

$\gamma = 7/5$

$P_r = 5/7$

$\zeta = 2/3$

$\dfrac{p_p}{p_{p_E}}$

Sym	p_o, torr	T_o, °K	T_w, °K	r_w, cm	z, cm	Source
O	0.359	717	85	30.5	96.5	Experimental Data
——	0.360	717	89	30.2	95.3	Present Calculation

1.0 0.8 0.6 0.4 0.2 0

0 0.2 0.4 0.6 0.8 1.0 0.8 0.6 0.4 0.2 0

y/r_w
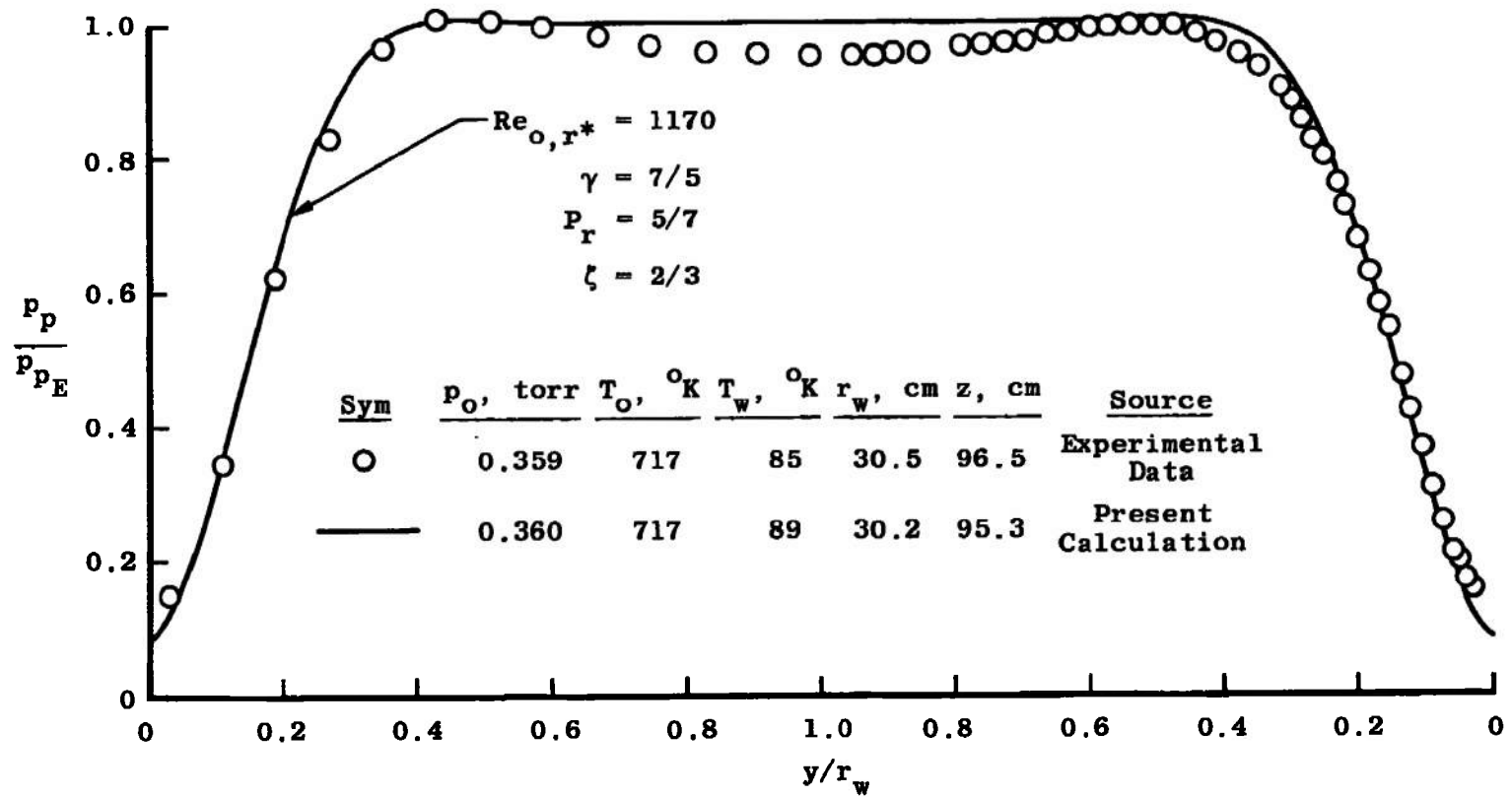
Fig. 18 Calculated and Measured Pitot Pressure Profiles Downstream
of the M3 Nozzle Throat for $\mathrm{Re}_{o,r*} = 1170$

Sym	p_o, torr	T_o, °K	T_w, °K	v_w, cm	z, cm	Source
O	0.103	717	85	30.5	96.5	Experimental Data
——	0.100	717	89	29.0	88.6	Present Calculation
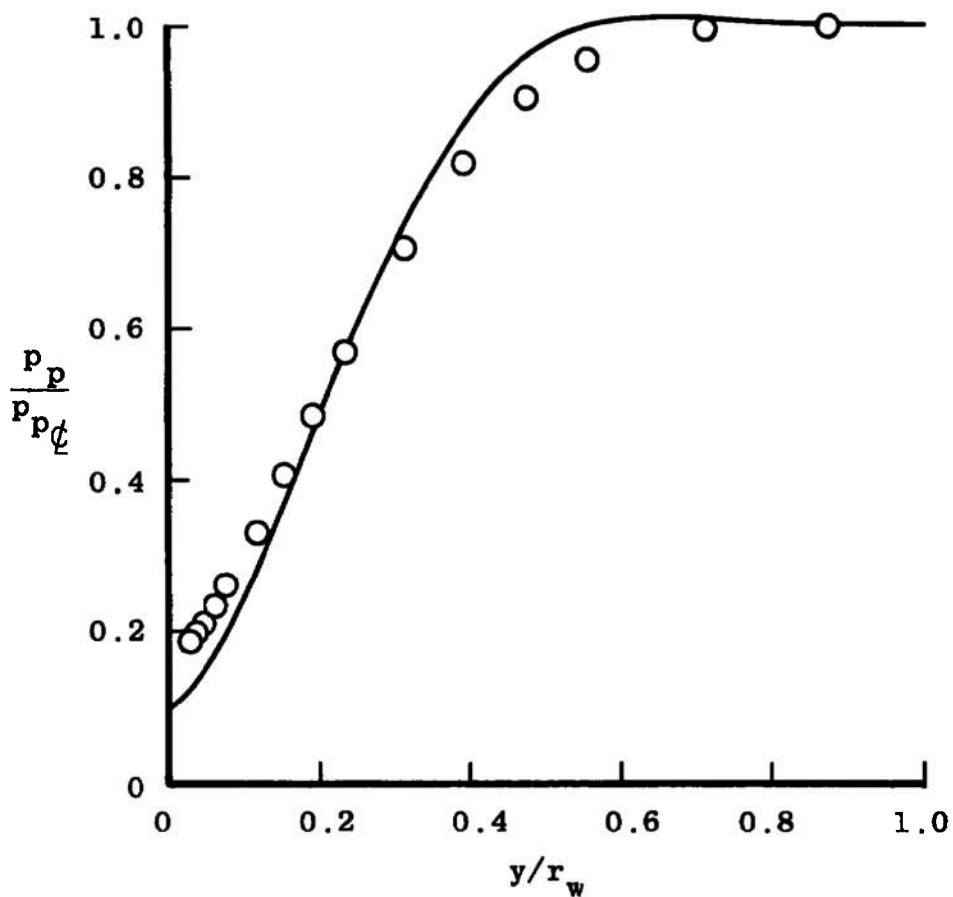
Fig. 19 Calculated and Measured Pitot Pressure Profiles Downstream of the M3 Nozzle Throat for $Re_{o,r\bullet} = 330$
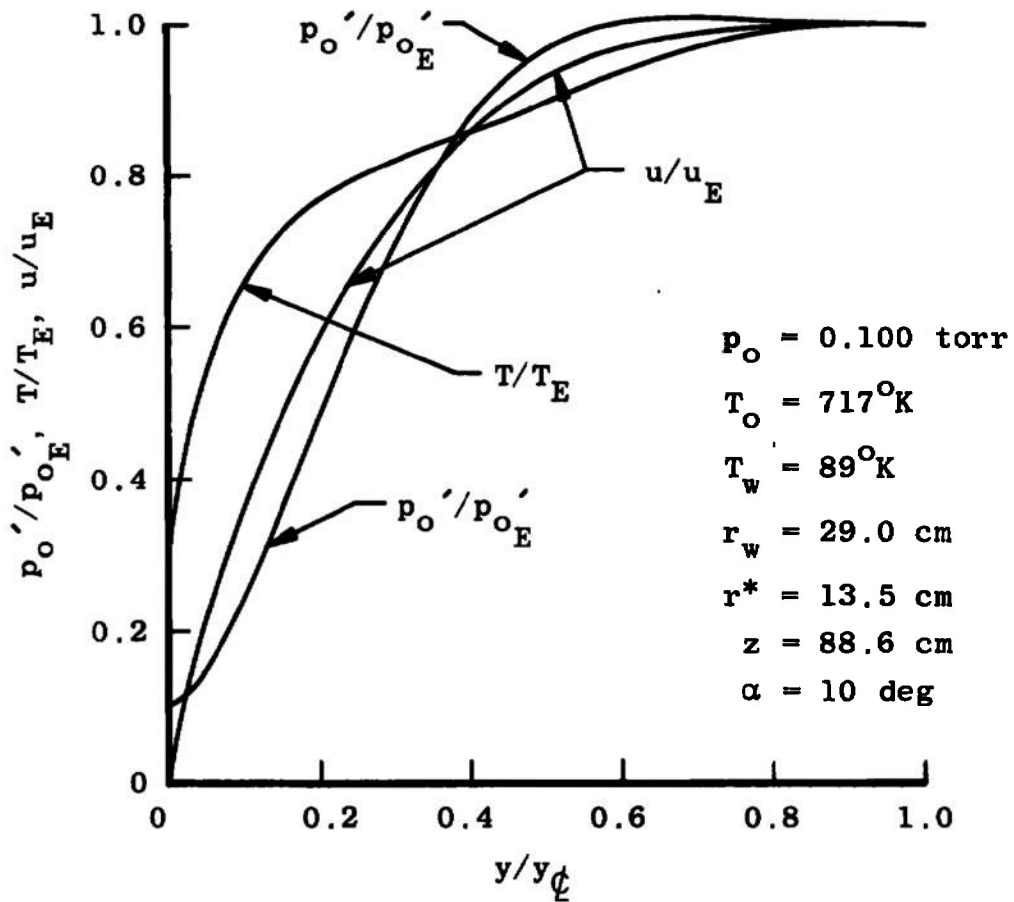
Fig. 20 Calculated Velocity, Temperature, and Pitot Pressure Profiles
Downstream of the M3 Nozzle Throat for $Re_{o,r*} = 330$

Curve	Numerical Method	Higher-Order Effects	Source
————	Beckwith and Cohen (Local Similarities, Ref. 17)	None	Ref. 16
— — — —	Incremental Flat Plate (Ref. 18)	None	Ref. 16
—— — — ——	Jaffe, Lind, and Smith (Nonsimilar, Ref. 7)	None	Ref. 5
——✗——✗——	Patankar and Spalding (Nonsimilar, Ref. 11)	Transverse Curvature	Present
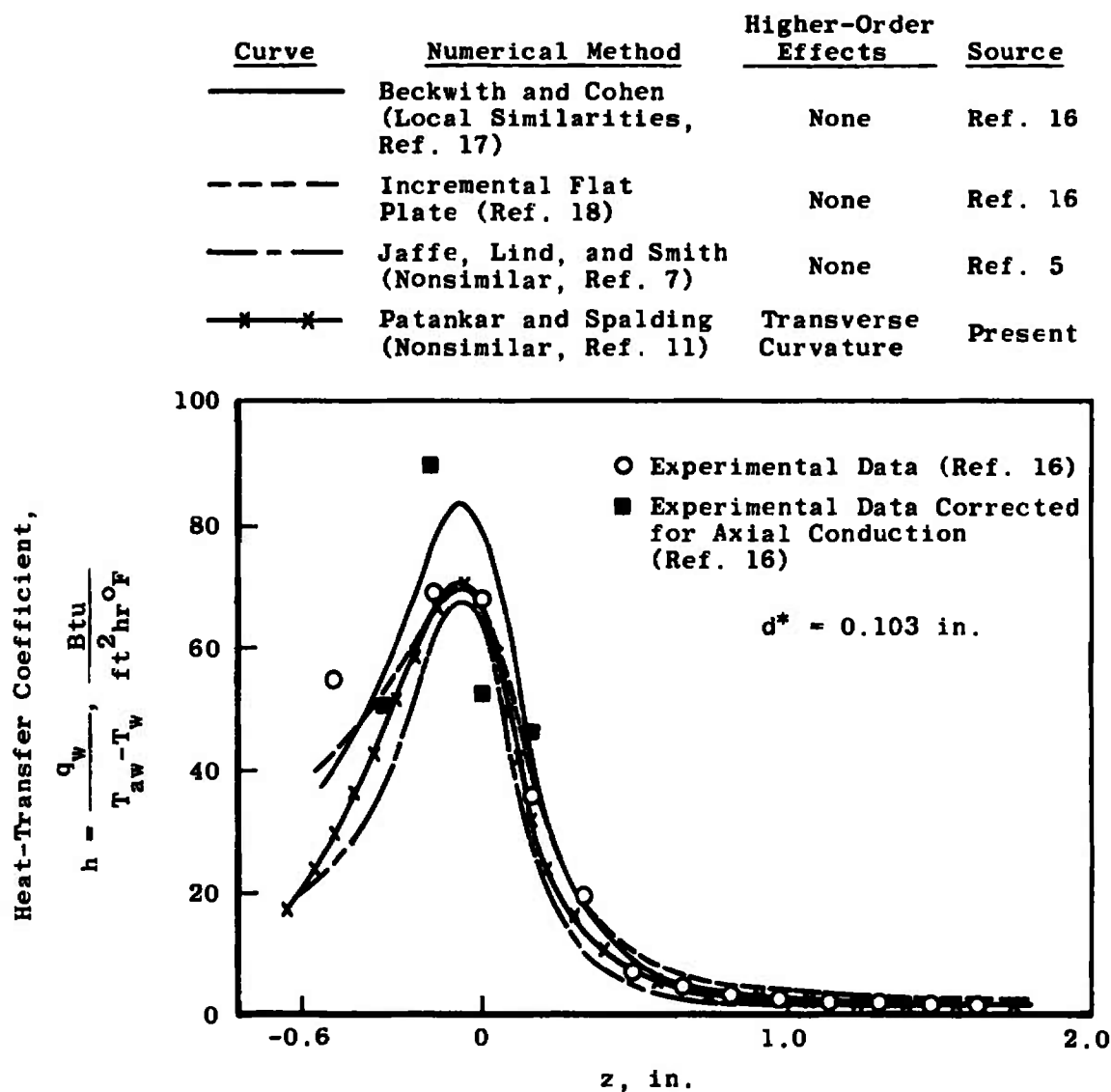
Fig. 21 Calculated and Measured Heat-Transfer Coefficients in a Low-Density Nozzle

TABLE I
CONDITIONS FOR THE NUMERICAL SOLUTIONS PRESENTED IN SECTION IV

Condition No.	γ	Pr	ζ	T_w/T_o	α, dcg	A/Λ^*	$\bar{R}$	Re_{o,r^*}	Comment
1	9/7	3/4	3/4	1/10	10	25	3	3×10^3, 10^4, 10^5	Axisymmetric Nozzle with Transverse Curvature
2	5/3	2/3	5/7	↓	10	↓	↓	5×10^3, 10^4, 2×10^4, 5×10^4, 10^5	↓
3	7/5	5/7	2/3		10			10^3, 3×10^3, 10^4, 3×10^4, 10^5	
4				↓	15			3×10^3, 10^4, 10^5	
5				1/3	10			5×10^3, 10^4, 3×10^4, 10^5	
6		↓		Adiabatic				10^4, 10^5	
7		1	↓	1/10				3×10^3, 10^4, 3×10^4, 10^5	
8		5/7	1			↓	↓	10^3, 10^4, 10^5	
9		5/7	2/3					3×10^3	Axisymmetric Nozzle without Transverse Curvature
10	↓	5/7	2/3	↓	↓	5	9	3×10^3	Two-Dimensional Nozzle

APPENDIX III
BOUNDARY-LAYER COMPUTER CODE

```
FORTRAN IV G LEVEL  20              MAIN          DATE = 72286       22/52/18
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```
           C       MAIN                                                    A   2
 0001              COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,   A   3
                  1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI   A   4
                  2MEN,IHEAT,Z,TD,XSTEP/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD/B/B   A   5
                  3ETA,GAMA(2),TAUI,TAUE,AJI(2),AJE(2),INDI(2),INDE(2)/V/U(200),F(2,2   A   6
                  400),R(200),RHO(200),OM(200),Y(200)/C/SC(200),AU(200),BU(200),CU(20   A   7
                  50),A(2,200),B(2,200),C(2,200)/O/YR(200),UR(200),RR(200),HR(200),XM   A   8
                  6(200),PITOT(200),TEMP(200)/E/DSTAR(300),XRS(300),RWRS(300),COSAL(3   A   9
                  700)                                                     A  10
 0002              COMMON /L/ AK,ALMG                                      A  11
 0003      1       CONTINUE                                                A  12
 0004              AK=1.                                                   A  13
 0005              INTG=0                                                  A  14
 0006              CALL BEGIN                                              A  15
 0007              CALL PRE (XU,DPDX)                                      A  16
 0008              XTHOAT=3.14159*XR/2.                                    A  17
 0009              AMI=0.                                                  A  18
 0010              AME=0.                                                  A  19
 0011              GO TO 3                                                 A  20
 0012      2       CALL READY                                             A  21
           C       THE FOLLOWING STATEMENT IS TO KILL SHOT IF PROBLEMS (LIKE MERGING)   A  22
           C       OCCUR IN SUBROUTINE READY                              A  23
 0013              IF (XD.GT.XL) GO TO 13                                 A  24
 0014      3       CONTINUE                                               A  25
 0015              INTG=INTG+1                                            A  26
           C       CHOICE OF FORWARD STEP                                 A  27
 0016              DX=XSTEP*R(1)                                          A  28
 0017              XD=XU+DX                                               A  29
 0018              IF (.NOT.(XU.LT.XTHOAT.AND.XD.GT.XTHOAT)) GO TO 4      A  30
 0019              XD=XTHOAT                                              A  31
 0020              DX=XD-XU                                               A  32
 0021              ITHOAT=INTG+1                                          A  33
 0022      4       CONTINUE                                               A  34
 0023              IF (XD.LE.XL) GO TO 5                                  A  35
 0024              XD=XL                                                  A  36
 0025              DX=XD-XU                                               A  37
 0026      5       CONTINUE                                               A  38
 0027              CALL PRE (XD,OPDX)                                     A  39
 0028              CALL ENTRN                                             A  40
 0029              CALL PRE (XU,DPDX)                                     A  41
 0030              IF (KASE.EQ.2) GO TO 6                                 A  42
 0031              IF (KIN.EQ.1) CALL MASS (XU,XD,AMI)                    A  43
 0032              IF (KEX.EQ.1) CALL MASS (XU,XD,AME)                    A  44
 0033              CALL WALL                                              A  45
 0034      6       CALL OUTPUT                                            A  46
 0035              CALL PRE (XD,OPDX)                                     A  47
 0036              CALL COEFF                                             A  48
           C       SETTING UP VELOCITIES AT A FREE BOUNDARY              A  49
           C       MODIFIED FOLLOWING STATEMENT FOR INTERNAL CORE FLOW    A  50
 0037              IF (KEX.EQ.2) U(NP3)=SQRT(2.)*SQRT(1.-PPO**((GAM-1.)/GAM))   A  51
 0038              IF (KIN.EQ.2) U(1)=SQRT(U(1)*U(1)-2.*(XD-XU)*OPOX/RHO(1))   A  52
 0039              CALL SOLVE (AU,BU,CU,U,NP3)                            A  53
           C       SETTING UP VELOCITIES AT A SYMMETRY LINE              A  54
 0040              IF (KIN.NE.3) GO TO 7                                  A  55
 0041              U(1)=U(2)                                              A  56
 0042              IF (KRAD.EQ.0) U(1)=.75*U(2)+.25*U(3)                  A  57
```

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FORTRAN IV G LEVEL  20              MAIN            DATE = 72286        22/52/18

  0043        7      IF (KEX.EQ.3) U(NP3)=.75*U(NP2)+.25*U(NP1)                    A  58
  0044               IF (NEQ.EQ.1) GO TO 14                                       A  59
  0045               DO 13 J=1,NPH                                                A  60
  0046               DO 8 I=2,NP2                                                 A  61
  0047        .      AU(I)=A(J,I)                                                 A  62
  0048               BU(I)=B(J,I)                                                 A  63
  0049        8      CU(I)=C(J,I)                                                 A  64
  0050               DO 9 I=1,NP3                                                 A  65
  0051        9      SC(I)=F(J,I)                                                 A  66
  0052        .      CALL SOLVE (AU,BU,CU,SC,NP3)                                 A  67
  0053               DO 10 I=1,NP3                                                A  68
  0054       10      F(J,I)=SC(I)                                                 A  69
  0055               IF (KASE.EQ.2) GO TO 11                                      A  70
             C       SETTING UP WALL VALUES OF F                                  A  71
  0056               IF (KIN.EQ.1.AND.INDI(J).EQ.2) F(J,1)=((1.+BETA+GAMA(J))*F(J,2)-(1 A  72
                    1.+BETA-GAMA(J))*F(J,3))*.5/GAMA(J)                           A  73
  0057               IF (KEX.EQ.1.AND.INDE(J).EQ.2) F(J,NP3)=((1.+BETA+GAMA(J))*F(J,NP2 A  74
                    1)-(1.+BETA-GAMA(J))*F(J,NP1))*.5/GAMA(J)                     A  75
             C       SETTING UP SYMMETRY-LINE VALUES OF F                         A  76
  0058       11      IF (KIN.NE.3) GO TO 12                                       A  77
  0059               F(J,1)=F(J,2)                                                A  78
  0060               IF (KRAD.EQ.0) F(J,1)=.75*F(J,2)+.25*F(J,3)                  A  79
  0061       12      IF (KEX.EQ.3) F(J,NP3)=.75*F(J,NP2)+.25*F(J,NP1)             A  80
  0062       13      CONTINUE                                                     A  81
  0063       14      XP=XU                                                        A  82
  0064               XRS(INTG)=XU                                                 A  83
  0065               RWPS(INTG)=R(1)                                             A  84
  0066               COSAL(INTG)=CSALFA                                           A  85
  0067               XU=XD                                                        A  86
  0068               PEI=PEI+DX*(R(1)*AMI-R(NP3)*AME)                             A  87
             C       THE TERMINATION CONDITION                                    A  88
  0069        .      IF (XU.LT.XL) GO TO 2                                        A  89
  0070               CALL NEWPPO                                                  A  90
  0071               STOP                                                         A  91
  0072               END                                                         A  92-
```

FORTRAN IV G LEVEL 20		NEWPPO	DATE = 72286	22/52/18		

```
0001            SUBROUTINE NEWPPO                                             B    1
0002            COMMON /GEN/ PEI,AMI,AME,DPDX,PREFI2I,PR(2I,PI2),DEN,AMU,XU,XD,XP,   B    2
               1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI   B    3
               2MEN,IHEAT,Z,TO,XSTEP/E/DSTAR(300),XRS(300),RWRS(300),COSAL(300)      B    4
0003            DIMENSION FMM(300), POP(300), RIRS(300)                       B    5
0004            IF (IDIMEN.EQ.0) RETURN                                       B    6
0005            G=GAM                                                         B    7
0006            C1=(G+1.)/2.                                                  B    8
0007            C2=(G-1.)/2.                                                  B    9
0008            C3=(G+1.)/(2.*(G-1.))                                         B   10
0009            C4=C1**C3                                                     B   11
0010            C5=C4*C2*C3*2.                                                B   12
0011            DO 1 I=1,INTG                                                 B   13
0012        1   RIRS(I)=RWRS(I)-COSAL(I)*DSTAR(I)                             B   14
0013            DO 2 I=1,INTG                                                 B   15
0014            IF (RIRS(I).LT.RIRS(I+1)) GO TO 3                             B   16
0015        2   CONTINUE                                                      B   17
0016        3   RTHOAT=RWRS(I)-COSAL(I)*DSTAR(I)                              B   18
0017            DO 14 I=1,INTG                                                B   19
0018            A=(RTHOAT/RIRS(I))**2                                         B   20
0019            IF (1-I) 4,4,6                                                B   21
0020        4   FM1=0.5                                                       B   22
0021        5   B=(1.+C2*FM1**2)                                             B   23
0022            FM=FM1-(A*B**(C3+1.)-C4*FM1*B)/(C5*FM1**2-C4*B)               B   24
0023            IF (ABS(FM-FM1)-.00001) 13,12,12                             B   25
0024        6   IF (A-1.0) 8,7,7                                             B   26
0025        7   FM=1.0                                                        B   27
0026            B=(1.+C2)                                                     B   28
0027            GO TO 13                                                      B   29
0028        8   CONTINUE                                                      B   30
0029            IF (FM-1.0) 9,10,10                                          B   31
0030        9   FM1=FM                                                        B   32
0031            GO TO 11                                                      B   33
0032       10   FM1=FM+0.1                                                    B   34
0033       11   CONTINUE                                                      B   35
0034            GO TO 5                                                       B   36
0035       12   FM1=FM                                                        B   37
0036            GO TO 5                                                       B   38
0037       13   POP(I)=B**(G/(1.-G))                                         B   39
0038            FMM(I)=FM                                                     B   40
0039       14   CONTINUE                                                      B   41
0040            WRITE (6,15)                                                  B   42
0041            WRITE (7,16) INTG                                             B   43
0042            WRITE (7,17) (XRS(I),POP(I),I=1,INTG)                         B   44
0043            WRITE (6,18) (FMM(I),POP(I),DSTAR(I),XRS(I),RWRS(I),I,I=1,INTG)  B   45
0044            RETURN                                                        B   46
       C                                                                      B   47
0045       15   FORMAT(1H1,2X,'THE DISPLACEMENT THICKNESS FROM THIS ITERATION AND  B   48
               1THE PRESSURE DISTRIBUTION FOR THE NEXT ITERATION FOLLOWS'//,  B   49
               28X,'M',11X,'P/PO',11X,'DS',13X,'X',12X,'RW',8X,'INTG'/)       B   50
0046       16   FORMAT(I3)                                                    B   51
0047       17   FORMAT(1P2E12.6,53X,I3)                                       B   52
0048       18   FORMAT(1P5E14.5,I6)                                           B   53
0049            END                                                          B   54
```

FORTRAN IV G LEVEL	20	BEGIN	DATE = 72286	22/52/18

```
0001              SUBROUTINE BEGIN                                                 C   1
0002              COMMON /GEN/ PEI,AMI,AME,DPOX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP, C   2
                 1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI  C   3
                 2MEN,IHEAT,Z,TO,XSTEP/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD/B/B  C   4
              .  3ETA,GAMA(2),TAUI,TAUE,AJI(2),AJE(2),INDI(2),INDE(2)/V/U(200),F(2,2  C   5
                 400),R(200),RHO(200),OM(200),Y(200)                                 C   6
              C   PROBLEM SPECIFICATION                                             C   7
0003              READ (5,28) KRAD,IDIMEN,NEQ,KEX,KIN,IHEAT,N                        C   8
0004              READ (5,29) REORS,ZETA,PR(1),GAM,ALPHA,XR,XL,USUP,YSTART,TWTO,XSTE  C   9
                 1P                                                                  C  10
              C   IDIMEN=0 FUR PLANAR FLOW AND IDIMEN=1 FOR AXISYMMETRIC FLOW         C  11
              C   INITIAL EDGE OF BOUNDARY LAYER IS YSTART                           C  12
0005              PREF(1)=PR(1)                                                      C  13
              C   APPROXIMATE CALCULATION OF UEDGE FROM ONE DIMENSION FLOW RELATIONS  C  14
0006              UUMXX= SQRT(2./(GAM-1.))*((GAM+1./2.)**((GAM+1.)/(2.*(GAM-1.)))    C  15
0007              UUMXX=(1./(1.+XR))**(1+IDIMEN)/UUMXX                               C  16
0008              UEDGE=UUMXX*SQRT(2.)                                              C  17
0009              KASE=2                                                            C  18
0010              IF (KIN.EQ.1.OR.KEX.EQ.1) KASE=1                                   C  19
0011              XU=0.                                                             C  20
0012              NPH=NEQ-1                                                          C  21
0013              NP1=N+1                                                            C  22
0014              NP2=N+2                                                            C  23
0015              NP3=N+3                                                            C  24
              C   INITIAL VELOCITY PROFILE                                          C  25
0016              Y(1)=0.0                                                           C  26
0017              U(1)=0.0                                                           C  27
0018              XNP2=NP2                                                           C  28
0019              DELY=YSTART/XNP2                                                   C  29
0020              DO 1 I=2,NP3                                                       C  30
0021              Y(I)=Y(I-1)+DELY                                                   C  31
0022            , ETA=Y(I)/(DELY*XNP2)                                              C  32
0023              U(I)=(2.*ETA-ETA**2)*UEDGE                                         C  33
0024      1       CONTINUE                                                          C  34
              C   CALCULATION OF SLIP VELOCITIES AND DISTANCES                       C  35
0025              BETA=1.0                                                           C  36
0026              GO TO (2,3,4), KIN                                                 C  37
0027      2       U(2)=U(3)/(1.+2.*BETA)                                             C  38
0028              Y(2)=Y(3)*BETA/(2.+BETA)                                           C  39
0029              GO TO 6                                                            C  40
0030      3       U11=U(1)*U(1)                                                      C  41
0031              U13=U(1)*U(3)                                                      C  42
0032              U33=U(3)*U(3)                                                      C  43
0033              SQ=84.*U11-12.*U13+9.*U33                                          C  44
0034              U(2)=(16.*U11-4.*U13+U33)/(2.*(U(1)+U(3))+SQRT(SQ))                C  45
0035              Y(2)=Y(3)*(U(2)+U(3)-2.*U(1))*.5/(U(2)+U(3)+U(1))                  C  46
0036              GO TO 6                                                            C  47
0037      4       IF (KRAD.NE.0) GO TO 5                                             C  48
0038              U(2)=(4.*U(1)-U(3))/3.                                             C  49
0039              Y(2)=0.                                                            C  50
0040              GO TO 6                                                            C  51
0041      5       U(2)=U(1)                                                          C  52
0042              Y(2)=Y(3)/3.                                                       C  53
0043      6       GO TO (7,8,9), KEX                                                 C  54
0044      7       U(NP2)=U(NP1)/(1.+2.*BETA)                                         C  55
0045              Y(NP2)=Y(NP3)-(Y(NP3)-Y(NP1))*BETA/(2.+BETA)                       C  56
```

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  0046                    GO TO 10                                                        C  57
  0047          8         U11=U(NP1)*U(NP1)                                               C  58
  0048                    U13=U(NP1)*U(NP3)                                               C  59
  0049                    U33=U(NP3)*U(NP3)                                               C  60
  0050          .         SQ=84.*U33-12.*U13+9.*U11                                       C  61
  0051                    U(NP2)=(16.*U33-4.*U13+U11)/(2.*(U(NP1)+U(NP3))+SQRT(SQ))       C  62
  0052                    Y(NP2)=Y(NP3)-(Y(NP3)-Y(NP1))*(U(NP2)+U(NP1)-2.*U(NP3))*.5/(U(NP2) C 63
                          1+U(NP1)+U(NP3))                                                C  64
  0053                    GO TO 10                                                        C  65
  0054          9    '    U(NP2)=(4.*U(NP3)-U(NP1))/3.                                    C  66
  0055                    Y(NP2)=Y(NP3)                                                   C  67
  0056         10         CONTINUE                                                        C  68
  0057                    IF (NEQ.EQ.1) GO TO 20                                          C  69
  0058                    DO 19 J=1,NPH                                                   C  70
               C          INITIAL PROFILES OF OTHER DEPENDENT VARIABLES                  C  71
  0059                    DO 11 I=1,NP3                                                   C  72
  0060                    ETA=Y(I)/(DELY*XNP2)                                            C  73
  0061                    F(J,I)=TWTO+(2.*ETA-ETA**2)*(1.-TWTO)                           C  74
  0062         11         CONTINUE                                                        C  75
               C          CALCULATION OF CORRESPONDING SLIP VALUES                        C  76
  0063                    GAMA(J)=1.0                                                     C  77
  0064                    GO TO (12,13,14), KIN                                           C  78
  0065         12         F(J,2)=F(J,1)+(F(J,3)-F(J,1))*(1.+BETA-GAMA(J))/(1.+BETA+GAMA(J)) C 79
  0066                    GO TO 15                                                        C  80
  0067         13         G=(U(2)+U(3)-8.*U(1))/(5.*(U(2)+U(3))+8.*U(1))                  C  81
  0068                    GF=(1.-PREF(J))/(1.+PREF(J))                                    C  82
  0069                    GF=(G+GF)/(1.+G*GF)                                             C  83
  0070                    F(J,2)=F(J,3)*GF+(1.-GF)*F(J,1)                                 C  84
  0071                    GO TO 15                                                        C  85
  0072         14         F(J,2)=F(J,1)                                                   C  86
  0073                    IF (KRAD.EQ.0) F(J,2)=(4.*F(J,1)-F(J,3))/3.                     C  87
  0074         15         GO TO (16,17,18), KEX                                           C  88
  0075         16         F(J,NP2)=F(J,NP3)+(F(J,NP1)-F(J,NP3))*(1.+BETA-GAMA(J))/(1.+BETA+G C 89
                          1AMA(J))                                                        C  90
  0076                    GO TO 19                                                        C  91
  0077         17         G=(U(NP2)+U(NP1)-8.*U(NP3))/(5.*(U(NP2)+U(NP1))+8.*U(NP3))      C  92
  0078                    GF=(1.-PREF(J))/(1.+PREF(J))                                    C  93
  0079                    GF=(G+GF)/(1.+G*GF)                                             C  94
  0080                    F(J,NP2)=F(J,NP1)*GF+(1.-GF)*F(J,NP3)                           C  95
  0081                    GO TO 19                                                        C  96
  0082         18         F(J,NP2)=(4.*F(J,NP3)-F(J,NP1))/3.                              C  97
  0083         19         CONTINUE                                                        C  98
  0084         20         CONTINUE                                                        C  99
  0085                    CALL DENSTY                                                     C 100
               C          CALCULATION OF RADII                                            C 101
  0086                    CALL RAD (XU,R(1),CSALFA)                                       C 102
  0087                    IF (CSALFA.EQ.0..OR.KRAD.EQ.0) GO TO 22                         C 103
  0088                    DO 21 I=2,NP3                                                   C 104
  0089         21         R(I)=R(1)-Y(I)*CSALFA                                           C 105
               C          CHANGE MADE IN STATEMENT NUMBER 28 FOR INTERNAL FLOW            C 106
  0090                    GO TO 24                                                        C 107
  0091         22         DO 23 I=2,NP3                                                   C 108
  0092         23         R(I)=R(1)                                                       C 109
  0093         24         CONTINUE                                                        C 110
               C          CALCULATION OF OMEGA VALUES                                     C 111
  0094                    OM(1)=0.                                                        C 112
```

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FORTRAN IV G LEVEL    20              BEGIN            DATE = 72286          22/52/18
  0095                    OM(2)=0.                                                      C 113
  0096                    DO 25 I=3,NP2                                                 C 114
  0097           25       OM(I)=OM(I-1)+.5*(RHO(I)*U(I)*R(I)+RHO(I-1)*U(I-1)*R(I-1))*(Y(I)-Y  C 115
                        1(I-1))                                                         C 116
  0098                    PEI=OM(NP2)                                                   C 117
  0099                    DO 26 I=3,NP1                                                 C 118
  0100           26       OM(I)=OM(I)/PEI                                               C 119
  0101                    OM(NP2)=1.                                                    C 120
  0102                    OM(NP3)=1.                                                    C 121
  0103                    IF (NEQ.EQ.1) RETURN                                          C 122
  0104                    DO 27 J=1,NPH                                                 C 123
  0105                    IF (KEX.EQ.1) INDE(J)=1                                       C 124
  0106                    IF (KIN.EQ.1) INDI(J)=1                                       C 125
  0107           27       CONTINUE                                                      C 126
  0108                    RETURN                                                        C 127
                  C                                                                     C 128
  0109           28       FORMAT(6I1,I3)                                                C 129
  0110           29       FORMAT (8E10.0)                                              C 130
  0111                    END                                                           C 131-
```

```
 0001                    SUBROUTINE COEFF                                      0    1
 0002                    COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,  D    2
                        1XL,OX,INTG,CSALFA,ALPHA,XP,REURS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI  D    3
                        2MEN,IHEAT,Z,TO,XSTEP/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD/B/B  0    4
                        3ETA,GAMA(2),TAUI,TAUE,AJI(2),AJE(2),INDI(2),INDE(2)/V/U(200),F(2,2  D    5
                        400),R(200),RHO(200),OM(200),Y(200)/C/SC(200),AU(200),BU(200),CU(20  D    6
                        50),A(2,200),B(2,200),C(2,200)                         D    7
 0003                    COMMON /L/ AK,ALMG                                     D    8
 0004                    DIMENSION G1(200), G2(200), G3(200), D(2,200), S1(200), S2(200), S  D    9
                        13(200)                                               D   10
          C             CALCULATION OF SMALL C 'S                              D   11
 0005                    DO 1 I=2,NP1                                          D   12
 0006                    RA=.5*(R(I+1)+R(I))                                   D   13
 0007                    RH=.5*(RHO(I+1)+RHO(I))                               D   14
 0008                    UM=.5*(U(I+1)+U(I))                                   0   15
 0009                    CALL VEFF (I,I+1,FMU)                                 D   16
 0010        1           SC(I)=RA*RA*RH*UM*EMU/(PEI*PEI)                       D   17
          C             THE CONVECTION TERM                                    D   18
 0011                    SA=R(1)*AMI/PEI                                       D   19
 0012                    SB=(R(NP3)*AME-R(1)*AMI)/PEI                          0   20
 0013                    DX=XD-XU                                              D   21
 0014                    DO 4 I=3,NP1                                          D   22
 0015                    OMD=OM(I+1)-OM(I-1)                                   D   23
 0016                    P2=.25/OX                                             D   24
 0017                    P3=P2/OMD                                             D   25
 0018                    P1=(OM(I+1)-OM(I))*P3                                 D   26
 0019                    P3=(OM(I)-OM(I-1))*P3                                 D   27
 0020                    P2=3.*P2                                              0   28
 0021                    Q=SA/OMD                                             D   29
 0022                    R2=-SB*.25                                            D   30
 0023                    R3=R2/OMD                                            0   31
 0024                    R1=-(OM(I+1)+3.*OM(I))*R3                             D   32
 0025                    R3=(OM(I-1)+3.*OM(I))*R3                              D   33
 0026                    G1(I)=P1+Q+R1                                         D   34
 0027                    G2(I)=P2+R2                                           D   35
 0028                    G3(I)=P3-Q+R3                                         D   36
 0029                    CU(I)=-P1*U(I+1)-P2*U(I)-P3*U(I-1)                    D   37
          C             THE DIFFUSION TERM                                     D   38
 0030                    AU(I)=2./OMD                                          D   39
 0031                    BU(I)=SC(I-1)*AU(I)/(OM(I)-OM(I-1))                   D   40
 0032                    AU(I)=SC(I)*AU(I)/(OM(I+1)-OM(I))                     D   41
 0033                    IF (NEQ.EQ.1) GO TO 3                                 D   42
 0034                    DO 2 J=1,NPH                                          D   43
 0035                    C(J,I)=-P1*F(J,I+1)-P2*F(J,I)-P3*F(J,I-1)             D   44
 0036                    CALL SOURCE (J,I,CS,D(J,I))                           D   45
 0037                    C(J,I)=-C(J,I)+CS-F(J,I)*D(J,I)                       D   46
 0038                    A(J,I)=AU(I)/PREF(J)                                  D   47
 0039                    B(J,I)=BU(I)/PREF(J)                                  D   48
 0040        2           CONTINUE                                             0   49
          C             SOURCE TERM FOR VELOCITY EQUATION                      0   50
 0041        3           S1(I)=DPDX*DX                                         D   51
 0042                    S2(I)=P2*S1(I)/(RHO(I)*U(I))                          D   52
 0043                    S3(I)=P3*S1(I)/(RHO(I-1)*U(I-1))                      D   53
 0044                    S1(I)=P1*S1(I)/(RHO(I+1)*U(I+1))                      D   54
 0045                    CU(I)=-CU(I)-2.*(S1(I)+S2(I)+S3(I))                   0   55
 0046                    S1(I)=S1(I)/U(I+1)                                    D   56
```

FORTRAN IV G LEVEL	20	COEFF	DATE = 72286	22/52/18		
0047		S2(I)=S2(I)/U(I)			D	57
0048		S3(I)=S3(I)/U(I-1)			0	58
0049	4	CONTINUE			0	59
	C	COEFFICIENTS IN THE FINAL FORM			D	60
0050		DO 5 I=3,NP1			D	61
0051		RL=1./(G2(I)+AU(I)+BU(I)-S2(I))			D	62
0052		AU(I)=(AU(I)+S1(I)-G1(I))*RL			D	63
0053		BU(I)=(BU(I)+S3(I)-G3(I))*RL			D	64
0054	5	CU(I)=CU(I)*RL			D	65
0055		IF (NEU.EQ.1) GO TO 7			0	66
0056		DO 6 J=1,NPH			0	67
0057		DO 6 I=3,NP1			D	68
0058		RL=1./(G2(I)+A(J,I)+B(J,I)-D(J,I))			D	69
0059		A(J,I)=(A(J,I)-G1(I))*RL			D	70
0060		B(J,I)=(B(J,I)-G3(I))*RL			D	71
0061	6	C(J,I)=C(J,I)*RL			D	72
0062	7	CALL SLIP			D	73
0063		RETURN			D	74
0064		END			0	75-

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FORTRAN IV G LEVEL  20              READY           DATE = 72286        22/52/18

  0001                  SUBROUTINE READY                                          E    1
  0002                  COMMON /GEN/ PEI,AMI,AME,OPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,  E    2
                       1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI  E    3
                       2MEN,IHEAT,Z,TU,XSTEP/V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2  E    4
                       300)/I/N,NP1,NP2,NP3,NEU,NPH,KEX,KIN,KASE,KRAD/B/BETA,GAMA(2),TAUI,  E    5
                       4TAUE,AJI(2),AJE(2),INDI(2),INDE(2)                          E    6
  0003                  CALL DENSTY                                               E    7
  0004                  CALL RAD (XU,R(1),CSALFA)                                  E    8
        C               Y NEAR THE I BOUNDARY                                     E    9
  0005           .      GO TO (1,2,3), KIN                                        E   10
  0006       1          Y(2)=(1.+BETA)*OM(3)*4./((3.*RHO(2)+RHO(3))*(U(2)+U(3)))   E   11
  0007                  GO TO 4                                                   E   12
  0008       2          Y(2)=12.*OM(3)/((3.*RHO(2)+RHO(3))*(U(2)+U(3)+4.*U(1)))    E   13
  0009                  GO TO 4                                                   E   14
  0010       3          Y(2)=.5*OM(3)/(RHO(1)*U(1))                               E   15
  0011       4          Y(3)=Y(2)+.25*OM(3)*(1./(RHO(3)*U(3))+2./(RHO(3)*U(3)+RHO(2)*U(2))  E   16
                       ))                                                         E   17
        C               Y 'S FOR INTERMEDIATE GRID POINTS                         E   18
  0012                  DO 5 I=4,NP1                                              E   19
  0013       5          Y(I)=Y(I-1)+.5*(OM(I)-OM(I-1))*(1./(RHO(I)*U(I))+1./(RHO(I-1)*U(I-  E   20
                       1)))                                                       E   21
        C               Y NEAR THE E BOUNDARY                                     E   22
  0014                  Y(NP2)=Y(NP1)+.25*(OM(NP2)-OM(NP1))*(1./(RHO(NP1)*U(NP1))+2./(RHO(  E   23
                       NP1)*U(NP1)+RHO(NP2)*U(NP2)))                              E   24
  0015           .      GO TO (6,7,8), KEX                                        E   25
  0016       6          Y(NP3)=Y(NP2)+(1.+BETA)*(OM(NP2)-OM(NP1))*4./((RHO(NP1)+3.*RHO(NP2  E   26
                       ))*(U(NP1)+U(NP2)))                                        E   27
  0017                  GO TO 9                                                   E   28
  0018       7          Y(NP3)=Y(NP2)+12.*(OM(NP2)-OM(NP1))/((RHO(NP1)+3.*RHO(NP2))*(U(NP2  E   29
                       )+U(NP1)+4.*U(NP3)))                                       E   30
  0019                  GO TO 9                                                   E   31
  0020       8          Y(NP3)=Y(NP2)+.5*(OM(NP2)-OM(NP1))/(RHO(NP3)*U(NP3))       E   32
  0021       9          IF (CSALFA.EQ.0..OR.KRAD.EQ.0) GO TO 11                    E   33
        C               XXX IS USED TO KILL SHOT IF NECESSARY                     E   34
  0022                  DO 10 I=2,NP3                                             E   35
  0023                  XXX=R(1)*R(1)-2.*Y(I)*PEI*CSALFA                          E   36
  0024                  IF (XXX.LT.0.0) XD=2.*XL                                  E   37
  0025                  IF (XXX.LT.0.0) GO TO 14                                  E   38
  0026      10          Y(I)=2.*Y(I)*PEI/(R(1)+SQRT(R(1)*R(1)-2.*Y(I)*PEI*CSALFA))  E   39
        C               CHANGED SIGN OF 2 IN THE DENOMINATOR OF ABOVE FOR INTERNAL FLOW  E   40
  0027                  GO TO 13                                                  E   41
  0028      11          DO 12 I=2,NP3                                             E   42
  0029      12          Y(I)=PEI*Y(I)/R(1)                                        E   43
  0030      13          Y(2)=2.*Y(2)-Y(3)                                         E   44
  0031                  Y(NP2)=2.*Y(NP2)-Y(NP1)                                   E   45
        C               CALCULATION OF RADII                                      E   46
  0032                  DO 14 I=2,NP3                                             E   47
  0033                  IF (KRAD.EQ.0) R(I)=R(1)                                  E   48
  0034                  IF (KRAD.NE.0) R(I)=R(1)-Y(I)*CSALFA                      E   49
        C               CHANGED SIGN IN EXPRESSION ABOVE FOR INTERNAL FLOW        E   50
  0035      14          CONTINUE                                                  E   51
  0036                  IF (R(NP3).LE.0.0) XD=2.*XL                               E   52
  0037                  IF (Y(NP3).LT.0.0) XD=2.*XL                               E   53
  0038                  RETURN                                                    E   54
  0039                  END                                                       E   55-
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FORTRAN IV G LEVEL   20              DENSTY           DATE = 72286        22/52/18

    0001                     SUBROUTINE DENSTY                                      F    1
    0002                     COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XO,XP,  F    2
                            1XL,DX,INTG,CSALFA,ALPHA,XR,REDRS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI  F    3
                            2MFN,IHEAT,Z,TO,XSTEP/V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2  F    4
                            300)/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD                        F    5
    0003                     TNP3=F(1,NP3)-U(NP3)**2/2.                                           F    6
    0004                     RHONP3=TNP3**(1./(GAM-1.))                                           F    7
    0005                     DO 1 I=1,NP3                                                         F    8
    0006                     T=F(1,I)-U(I)**2/2.                                                  F    9
    0007          1          RHO(I)=RHONP3*TNP3/T                                                 F   10
    0008                     RETURN                                                               F   11
    0009                     END                                                                  F   12-
```

```
FORTRAN IV G LEVEL   20              ENTRN            DATE = 72286        22/52/18

    0001                     SUBROUTINE ENTRN                                       G    1
    0002                     COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XO,XP,  G    2
                            1XL,DX,INTG,CSALFA,ALPHA,XR,REDRS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI  G    3
                            2MFN,IHEAT,Z,TO,XSTEP/V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2  G    4
                            300)/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD                        G    5
    0003          1          GO TO (2,3,6), KEX                                                   G    6
    0004          2          RETURN                                                               G    7
    0005          3          CONTINUE                                                             G    8
                  C          THE FOLLOWING AME IS FOR LAMINAR FLOW                                G    9
    0006                     IF (INTG.NE.1) GO TO 5                                               G   10
    0007                     DO 4 I=1,NP3                                                         G   11
    0008                     IF (OM(I).GT.0.9) N9=I                                               G   12
    0009                     IF (OM(I).GT.0.9) I=NP3                                              G   13
    0010          4          CONTINUE                                                             G   14
    0011                     DOM=OM(N9)-OM(N9-1)                                                  G   15
    0012                     DOM9=OM(N9)-0.9                                                      G   16
    0013          5          CONTINUE                                                             G   17
    0014                     U9=U(N9)-DOM9/DOM*(U(N9)-U(N9-1))                                    G   18
    0015                     TERMB=(U(N9)-U(N9-1))/DOM                                            G   19
    0016                     R9=R(N9)-DOM9/DOM*(R(N9)-R(N9-1))                                    G   20
    0017                     RHO9=RHO(N9)-DOM9/DOM*(RHO(N9)-RHO(N9-1))                            G   21
    0018                     VIS9=VISCO(N9)-DOM9/DOM*(VISCO(N9)-VISCO(N9-1))                      G   22
    0019                     CUUP=R(N9)*R(N9)*RHO(N9)*U(N9)*VISCO(N9)                             G   23
    0020                     CUUP=CUUP+R9*R9*RHO9*U9*VIS9                                         G   24
    0021                     CUUP=CUUP/(2.*PEI)                                                   G   25
    0022                     CUUM=R(N9-1)*R(N9-1)*RHO(N9-1)*U(N9-1)*VISCO(N9-1)                   G   26
    0023                     CUUM=CUUM+R9*R9*RHO9*U9*VIS9                                         G   27
    0024                     CUUM=CUUM/(2.*PEI)                                                   G   28
    0025                     G5=2.*CUUP/(DOM*DOM9)                                                G   29
    0026                     G6=2.*CUUM/(DOM*(0.9-OM(N9-1)))                                      G   30
    0027                     TERMA=G5*(U(N9)-U9)-G6*(U9-U(N9-1))                                  G   31
    0028          7          TERMA=TERMA/TERMB                                                    G   32
    0029                     U9WIG=USUP*SQRT(2.)*SQRT(1.-PPO**((GAM-1.)/GAM))                     G   33
                  C          USUP IS READ-IN IN BEGIN     IT SUPPRESSES THE B.L.                  G   34
    0030                     TERMC=(U9WIG-U9)/DX+DPDX/(RHO9*U9)                                   G   35
    0031                     TERMC=TERMC*PEI/TERMB                                                G   36
    0032                     AME=TERMA-TERMC                                                      G   37
    0033                     AMF=AME-0.1*R(1)*AMI                                                 G   38
    0034                     AME=AME/(0.9*R(NP3))                                                 G   39
    0035                     RETURN                                                               G   40
    0036          6          AME=0.                                                               G   41
    0037                     RETURN                                                               G   42
    0038                     END                                                                  G   43-
```

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FORTRAN IV G LEVEL   20              FBC          DATE = 72286        22/52/18

  0001              SUBROUTINE FBC (X,J,IND,AJFS)                              H    1
  0002              COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,  H    2
                   1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,T,TO,YSTART,USUP,IDI  H    3
                   2MEN,IHEAT,Z,TO,XSTEP                                       H    4
            C       TW IS PRESCRIBED IF IHEAT = 1 -- QODY IS PRESCRIBED IF NOT 1  H    5
  0003              IND=1                                                       H    6
  0004              AJFS=TWTO                                                   H    7
  0005              IF (IHEAT.EQ.1) GO TO 1                                     H    8
  0006              IND=2                                                       H    9
  0007              AJFS=0.0                                                    H   10
  0008        1     CONTINUE                                                    H   11
  0009              RETURN                                                      H   12
  0010              END                                                        H   13-
```

```
FORTRAN IV G LEVEL   20              MASS         DATE = 72286        22/52/18

  0001              SUBROUTINE MASS (XU,XD,AM)                                  I    1
            C       APPLICABLE TO AN IMPERMEABLE-WALL SITUATION                 I    2
  0002              AM=0.                                                       I    3
  0003              RETURN                                                      I    4
  0004              END                                                        I    5-
```

FURTRAN IV G LEVEL	20		OUTPUT	DATE = 72286	22/52/18		
0001			SUBROUTINE OUTPUT			J	1
0002			COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,			J	2
			1XL,DX,INTG,CSALFA,ALPHA,Xk,REORS,GAM,ZETA,PPU,TWTU,YSTART,USUP,IDI			J	3
			2MFN,IHEAT,Z,TU,XSTEP/V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2			J	4
			300)/C/SC(200),AU(200),BU(200),CU(200),A(2,200),B(2,200),C(2,200)/D			J	5
			4/YP(200),UR(200),RR(200),HR(200),XM(200),PITOT(200),TEMP(200)/E/DS			J	6
			5TAR(300),XRS(300),RWRS(300),COSAL(300)/I/N,NP1,NP2,NP3,NEQ,NPH,KEX			J	7
			6,KIN,KASE,KRAD/B/BETA,GAMA(2),TAUI,TAUE,AJI(2),AJE(2),INDI(2),INDE			J	8
			7(2)			J	9
0003			IF (INTG.NE.1) GO TO 1			J	10
0004			AL=ALPHA*180./3.14159265			J	11
0005			WRITE (6,7) (OM(I),I=1,NP3)			J	12
0006			WRITE (6,6) KRAD,IDIMEN,NEQ,KEX,KIN,IHEAT,N			J	13
0007	1		CONTINUE			J	14
0008			DPDXG=DPDX*GAM/(GAM-1.)			J	15
0009			IF (KRAD.EQ.0) DSTAR(INTG)=Y(NP3)-PEI/(R(1)*RHO(NP3)*U(NP3))			J	16
0010			IF (KRAD.EQ.0) GU TO 2			J	17
0011			DSTAR(INTG)=R(1)-SQRT(R(1)**2-2.*CSALFA*(R(1)*Y(NP3)-0.5*CSALFA*Y(			J	18
			1NP3)**2-PEI/(RHO(NP3)*U(NP3))))			J	19
0012			IF (CSALFA.NE.0.0) DSTAR(INTG)=DSTAR(INTG)/CSALFA			J	20
0013	2		CONTINUE			J	21
0014			IF (FLOAT(INTG-1)/5..NE.FLOAT((INTG-1)/5)) RETURN			J	22
0015			DU 3 I=1,NP3			J	23
0016			TEMP(I)=F(1,I)-U(I)**2/2.			J	24
0017			XM(I)=U(I)/SQRT((GAM-1.)*TEMP(I))			J	25
0018			XSQ=XM(I)**2			J	26
0019			IF (XM(I).LE.1.0) PITOT(I)=(1.+(GAM-1.)*XSQ/2.)**(GAM/(GAM-1.))			J	27
0020			IF (XM(I).LE.1.0) GO TO 3			J	28
0021			PITOT(I)=((GAM+1.)*XSQ/2.)**(GAM/(GAM-1.))			J	29
0022			PITOT(I)=PITOT(I)*((GAM+1.)/(2.*GAM*XSQ-GAM+1.))**(1./(GAM-1.))			J	30
0023	3		CONTINUE			J	31
0024			SQ2=SQRT(2.)			J	32
0025			DU 4 I=1,NP3			J	33
0026			PITOT(I)=PITOT(I)/PITOT(NP3)			J	34
0027			YR(I)=Y(I)*CSALFA/R(I)			J	35
0028			UR(I)=U(I)/U(NP3)			J	36
0029			RR(I)=RHO(I)/RHO(NP3)			J	37
0030			HR(I)=U(I)/SQ2			J	38
0031	4		CONTINUE			J	39
0032			WRITE (6,8)			J	40
0033			WRITE (6,12) INTG,XU,Z,REORS,GAM,PR(1),ZETA,TWTO,AL,XR,XL			J	41
0034			WRITE (6,13) NP3,DSTAR(INTG),PEI,AME,DPDXG,DX,CSALFA,USUP,PPO,TAUI			J	42
			1,AJI(1)			J	43
0035			WRITE (6,9)			J	44'
0036			DO 5 I=1,NP3,2			J	45
0037			WRITE (6,11) (Y(I),F(1,I),UR(I),HR(I),R(I),RHO(I),YR(I),XM(I),TEMP			J	46
			1(I),RR(I),PITOT(I))			J	47
0038	5		CONTINUE			J	48
0039			WPITE (6,10) (Y(NP3),F(1,NP3),UR(NP3),HR(NP3),R(NP3),RHO(NP3),YR(N			J	49
			1P3),XM(NP3),TEMP(NP3),RR(NP3),PITOT(NP3))			J	50
0040			WRITE (6,9)			J	51
0041			RETURN			J	52
	C					J	53
0042	6		FORMAT(////////,20X,'THE INPUT FLAGS ARE',///,25X,'KRAD = ',I1,			J	54
			1//,25X,'IDIMEN = ',I1,//,25X,'NEQ = ',I1,//,25X,'KEX = ',I1,			J	55
			2//,25X,'KIN = ',I1,//,25X,'IHEAT = ',I1,//,25X,'N = ',I3)			J	56

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FORTRAN IV G LEVEL   20              OUTPUT           DATE = 72286        22/52/18

  0043        7      FORMAT(24H1THE VALUES OF OMEGA ARE/(1P10E11.4))          J  57
  0044        8      FORMAT(1H1,4X,'INTG',9X,'XU',11X,'Z',9X,'REORS',7X,'GAMMA',8X,'PR'  J  58
                    1,9X,'ZETA',8X,'TW/TO',7X,'ALPHA',8X,'XR',10X,'XL'/,5X,'NP3',9X,'DS  J  59
                    2TAR',8X,'PEI',9X,'AME',8X,'DP/DX',8X,'DX',8X,'COSALF',7X,'USUP',8X  J  60
                    3,'P/PO',8X,'TAUI',7X,'AJI(1)'/)                          J  61
  0045        9      FORMAT(6X,'Y',9X,'H/HO',9X,'U/UE',8X,'U/UM',9X,'R',10X,'RHO',10X,  J  62
                    1'Y/R',9X,'M',11X,'T',10X,'N/NE',7X,'PITOT'/)             J  63
  0046       10      FORMAT(1P11E12.5,/)                                      J  64
  0047       11      FORMAT(1P11E12.5)                                        J  65
  0048       12      FORMAT(I8,4X,1P10E12.4)                                  J  66
  0049       13      FORMAT(I8,4X,1P10E12.4,//)                               J  67
  0050              END                                                       J  68-


FORTRAN IV G LEVEL   20               PRE             DATE = 72286        22/52/18

  0001              SUBROUTINE PRE (X,DPDXX)                                   K   1
  0002              COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,  K   2
                   1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI  K   3
                   2MEN,IHEAT,Z,TO,XSTEP,V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2  K   4
                   300)/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD              K   5
  0003              DIMENSION XX(300), POP(300)                               K   6
  0004              IF (INTG.NE.0) GO TO 1                                    K   7
  0005              READ (5,3) LMAX                                          K   8
  0006              READ (5,4) (XX(L),POP(L),L=1,LMAX)                       K   9
  0007              WRITE (6,5)                                             K  10
  0008              WRITE (6,6) (XX(L),POP(L),L,L=1,LMAX)                   K  11
  0009        1     CONTINUE                                                K  12
  0010              L=1                                                     K  13
  0011        2     CONTINUE                                                K  14
  0012              L=L+1                                                   K  15
  0013              IF (XX(L).LT.X) GO TO 2                                 K  16
  0014              DPDX=(POP(L)-POP(L-1))/(XX(L)-XX(L-1))                  K  17
  0015              PPO=POP(L-1)+DPDX*(X-XX(L-1))                           K  18
  0016              DPDXX=DPDX                                              K  19
  0017              DPDXX=DPDX*(GAM-1.)/GAM                                 K  20
  0018              RETURN                                                  K  21
             C                                                              K  22
  0019        3     FORMAT(I3)                                              K  23
  0020        4     FORMAT(2E12.0)                                          K  24
  0021        5     FORMAT(1H1,/,8X,'X',12X,'P/PO',/)                       K  25
  0022        6     FORMAT(1P2E14.5,I6)                                     K  26
  0023              END                                                     K  27-
```

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```
FORTRAN IV G LEVEL  20              RAD              DATE = 72286         22/52/18

  0001                 SUBROUTINE RAD (X,R1,CALPHA)                              L    1
  0002                 COMMON /GEN/ PEI,AHI,ANE,DPDX,PREF(2),PK(2),P(2),DEN,AMU,XU,XD,XP,  L    2
                      1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPU,TWTO,YSTART,USUP,IDI  L    3
                      2MEN,IHEAT,Z,TU,XSTEP/V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2  L    4
                      3001/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD              L    5
        C             APPLICABLE TO NOZZLES WITH CONSTANT LONGITUDINAL RADIUS OF  L    6
        C             CURVATURE OF THE CONVERGING SECTION- CONSTANT WALL HALF ANGLE OF  L    7
        C             DIVERGING SECTION-AND WALL SLOPES MATCHED DOWNSTREAM OF THE THROAT  L    8
  0003                 IF (INTG.NE.0) GO TO 1                                     L    9
  0004                 PI2=3.14159265/2.                                         L   10
  0005                 ALPHA=ALPHA*PI2/90.                                       L   11
  0006                 COSALF=COS(ALPHA)                                         L   12
  0007                 SINALF=SIN(ALPHA)                                         L   13
  0008                 ZWIG=XR*(1.+SINALF)                                       L   14
  0009                 XWIG=XR*(PI2+ALPHA)                                       L   15
  0010                 RWIG=1.+XR*(1.-COSALF)                                    L   16
  0011          1      CONTINUE                                                  L   17
  0012                 IF (X.GE.XWIG) GO TO 2                                     L   18
  0013                 R(1)=1.+XR*(1.-SIN(X/XR))                                 L   19
  0014                 CSALFA=SIN(X/XR)                                          L   20
  0015                 CALPHA=CSALFA                                             L   21
  0016                 R1=R(1)                                                   L   22
  0017                 ZZ=XR*(1.-COS(X/XR))                                      L   23
  0018                 GO TO 3                                                   L   24
  0019          2      CONTINUE                                                  L   25
  0020                 R(1)=(X-XWIG)*SINALF+RWIG                                 L   26
  0021                 CSALFA=COSALF                                             L   27
  0022                 CALPHA=CSALFA                                             L   28
  0023                 R1=R(1)                                                   L   29
  0024                 ZZ=(X-XWIG)*CSALFA+ZWIG                                   L   30
  0025          3      CONTINUE                                                  L   31
  0026                 IF(IDIMEN.EQ.0)R1=1.                                      L
  0027                 ZZ=ZZ-XR                                                  L   32
  0028                 Z=ZZ                                                      L   33
  0029                 RETURN                                                    L   34
  0030                 END                                                       L   35-
```

FORTRAN IV G LEVEL 20		SLIP	DATE = 72286	22/52/18		

```
0001              SUBROUTINE SLIP                                          M    1
0002              COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,  M    2
            1XL,DX,INTG,CSALFA,ALPHA,XR,REDRS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI  M    3
            2MEN,IHEAT,Z,TO,XSTEP/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD/V/U  M    4
            3(200),F(2,200),R(200),RHO(200),DM(200),Y(200)/B/BETA,GAMA(2),TAUI,  M    5
            4TAUE,AJI(2),AJE(2),INDI(2),INDE(2)                               M    6
0003              COMMON /L/ AK,ALMG/C/ SC(200),AU(200),BU(200),CU(200),A(2,200),B(2,  M    7
            1200),C(2,200)                                                    M    8
           C      SLIP COEFFICIENTS NEAR THE I BOUNDARY FOR VELOCITY EQUATION  M    9
0004              CU(2)=0.                                                    M   10
0005              CU(NP2)=0.                                                  M   11
0006              GO TO (1,2,3), KIN                                          M   12
0007        1     BU(2)=0.                                                    M   13
0008              AU(2)=1./(1.+2.*BETA)                                       M   14
0009              GO TO 5                                                     M   15
0010        2     SQ=84.*U(1)*U(1)-12.*U(1)*U(3)+9.*U(3)*U(3)                 M   16
0011              BU(2)=8.*(2.*U(1)+U(3))/(2.*U(1)+7.*U(3)+SQRT(SQ))          M   17
0012              AU(2)=1.-BU(2)                                             M   18
0013              GO TO 5                                                     M   19
0014        3     BU(2)=0.                                                    M   20
0015              CALL VEFF (2,3,EMU)                                         M   21
0016              AK1=1./DX-DPDX/(RHO(1)*U(1)*U(1))                          M   22
0017              AK2=-U(1)*AK1+DPDX/(RHO(1)*U(1))                           M   23
0018              AJ=RHO(1)*U(1)*.25*(Y(2)+Y(3))**2/EMU                      M   24
0019              IF (KRAD.EQ.0) GO TO 4                                      M   25
0020              AU(2)=2./(2.+AJ*AK1)                                        M   26
0021              CU(2)=-.5*AJ*AK2*AU(2)                                      M   27
0022              GO TO 5                                                     M   28
0023        4     CU(2)=1./(2.+3.*AJ*AK1)                                     M   29
0024              AU(2)=CU(2)*(2.-AJ*AK1)                                     M   30
0025              CU(2)=-CU(2)*4.*AJ*AK2                                      M   31
           C      SLIP COEFFICIENTS NEAR THE E BOUNDARY FOR VELOCITY EQUATION  M   32
0026        5     GO TO (6,7,8), KEX                                          M   33
0027        6     AU(NP2)=0.                                                  M   34
0028              BU(NP2)=1./(1.+2.*BETA)                                     M   35
0029              GO TO 9                                                     M   36
0030        7     SQ=84.*U(NP3)*U(NP3)-12.*U(NP3)*U(NP1)+9.*U(NP1)*U(NP1)     M   37
0031              AU(NP2)=8.*(2.*U(NP3)+U(NP1))/(2.*U(NP3)+7.*U(NP1)+SQRT(SQ)) M   38
0032              BU(NP2)=1.-AU(NP2)                                          M   39
0033              GO TO 9                                                     M   40
0034        8     AU(NP2)=0.                                                  M   41
0035              CALL VEFF (NP1,NP2,EMU)                                     M   42
0036              BK1=1./DX-DPDX/(RHO(NP3)*U(NP3)*U(NP3))                     M   43
0037              BK2=-U(NP3)*BK1+DPDX/(RHO(NP3)*U(NP3))                      M   44
0038              BJ=RHO(NP3)*U(NP3)*.25*(2.*Y(NP3)-Y(NP1)-Y(NP2))**2/EMU     M   45
0039              CU(NP2)=1./(2.+3.*BJ*BK1)                                   M   46
0040              BU(NP2)=CU(NP2)*(2.-BJ*BK1)                                 M   47
0041              CU(NP2)=-CU(NP2)*4.*BJ*BK2                                  M   48
0042        9     IF (NEQ.EQ.1) RETURN                                        M   49
           C      SLIP COEFFICIENTS NEAR THE I BOUNDARY FOR OTHER EQUATIONS   M   50
0043              DO 20 J=1,NPH                                               M   51
0044              C(J,2)=0.                                                   M   52
0045              C(J,NP2)=0.                                                 M   53
0046              GO TO (10,12,13), KIN                                       M   54
0047        10    CALL FBC (XD,J,INDI(J),QI)                                  M   55
0048              IF (INDI(J).EQ.1) GO TO 11                                  M   56
```

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FORTRAN IV G LEVEL   20              SLIP              DATE = 72286        22/52/18

   0049                    AJI(J)=QI                                                M   57
   0050                    A(J,2)=1.                                                 M   58
   0051                    B(J,2)=0.                                                 M   59
   0052                    C(J,2)=8.*(1.+2.*BETA)*PREF(J)*AJI(J)/(AK*AK*BETA*(1.+BE  M   60
                          1TA)*(3.*RHO(2)+RHO(3))*U(3))                              M   61
   0053                    GO TO 15                                                  M   62
   0054          11        F(J,1)=QI                                                 M   63
   0055                    A(J,2)=(1.+BETA-GAMA(J))/(1.+BETA+GAMA(J))                M   64
   0056                    B(J,2)=1.-A(J,2)                                          M   65
   0057                    GO TO 15                                                  M   66
   0058          12        A(J,2)=(U(2)+U(3)-8.*U(1))/(5.*(U(2)+U(3))+8.*U(1))       M   67
   0059                    GF=(1.-PREF(J))/(1.+PREF(J))                              M   68
   0060                    A(J,2)=(A(J,2)+GF)/(1.+A(J,2)*GF)                         M   69
   0061                    B(J,2)=1.-A(J,2)                                          M   70
   0062                    GO TO 15                                                  M   71
   0063          13        B(J,2)=0.                                                 M   72
   0064                    CALL SOURCE (J,1,CS,DS)                                   M   73
   0065                    AK1=1./DX-DS                                              M   74
   0066                    AK2=-AK1*F(J,1)-CS                                        M   75
   0067                    AJF=AJ*PREF(J)                                            M   76
   0068                    IF (KRAD.EQ.0) GO TO 14                                   M   77
   0069                    A(J,2)=2./(2.+AJF*AK1)                                    M   78
   0070                    C(J,2)=-.5*AJF*AK2*A(J,2)                                 M   79
   0071                    GO TO 15                                                  M   80
   0072          14        C(J,2)=1./(2.+3.*AJF*AK1)                                 M   81
   0073                    A(J,2)=C(J,2)*(2.-AJF*AK1)                                M   82
   0074                    C(J,2)=-C(J,2)*4.*AJF*AK2                                 M   83
              C           SLIP COEFFICIENTS NEAR THE E BOUNDARY FOR OTHER EQUATIONS  M   84
   0075          15        GO TO (16,18,19), KEX                                     M   85
   0076          16        CALL FRC (XD,J,INDE(J),QE)                                M   86
   0077                    IF (INDE(J).EQ.1) GO TO 17                                M   87
   0078          1         AJE(J)=JE                                                 M   88
   0079                    B(J,NP2)=1.                                               M   89
   0080                    A(J,NP2)=0.                                               M   90
   0081                    C(J,NP2)=-8.*(1.+2.*BETA)*PREF(J)*AJE(J)/(AK*AK*BETA*(1.+BETA)*(1.  M   91
                          1.+BETA)*(RHO(NP1)+3.*RHO(NP2))*U(NP1))                    M   92
   0082                    GO TO 20                                                  M   93
   0083          17        F(J,NP3)=QE                                               M   94
   0084                    B(J,NP2)=(1.+BETA-GAMA(J))/(1.+BETA+GAMA(J))              M   95
   0085                    A(J,NP2)=1.-B(J,NP2)                                      M   96
   0086                    GO TO 20                                                  M   97
   0087          18        B(J,NP2)=(U(NP2)+U(NP1)-8.*U(NP3))/(5.*(U(NP2)+U(NP1))+8.*U(NP3))  M   98
   0088                    GF=(1.-PREF(J))/(1.+PREF(J))                              M   99
   0089                    B(J,NP2)=(B(J,NP2)+GF)/(1.+B(J,NP2)*GF)                   M  100
   0090                    A(J,NP2)=1.-B(J,NP2)                                      M  101
   0091                    GO TO 20                                                  M  102
   0092          19        A(J,NP2)=0.                                              M  103
   0093                    CALL SOURCE (J,NP3,CS,DS)                                 M  104
   0094                    BK1=1./DX-DS                                              M  105
   0095                    BK2=-BK1*F(J,NP3)-CS                                      M  106
   0096                    BJF=BJ*PREF(J)                                            M  107
   0097                    C(J,NP2)=1./(2.+3.*BJF*BK1)                               M  108
   0098                    B(J,NP2)=C(J,NP2)*(2.-BJF*BK1)                            M  109
   0099                    C(J,NP2)=-C(J,NP2)*4.*BJF*BK2                             M  110
   0100          20        CONTINUE                                                  M  111
   0101                    RETURN                                                    M  112
```

```
FORTRAN IV G LEVEL  20        SLIP       DATE = 72286    22/52/18
 0102            END                                            M 113-
```

```
FORTRAN IV G LEVEL  20        SOLVE      DATE = 72286    22/52/18
 0001          SUBROUTINE SOLVE (A,B,C,F,NP3)                   N   1
        C      THIS SOLVES EQUATIONS OF THE FORM                N   2
        C      F(I) = A(I)*F(I+1) + B(I)*F(I-1) + C(I)          N   3
        C      FOR I=2,NP2                                      N   4
 0002          DIMENSION A(NP3), B(NP3), C(NP3), F(NP3)         N   5
 0003          NP2=NP3-1                                        N   6
 0004          B(2)=B(2)*F(1)+C(2)                              N   7
 0005          DO 1 I=3,NP2                                     N   8
 0006          T=1./(1.-B(I)*A(I-1))                            N   9
 0007          A(I)=A(I)*T                                      N  10
 0008        1 B(I)=(B(I)*B(I-1)+C(I))*T                        N  11
 0009          DO 2 I=2,NP2                                     N  12
 0010          J=NP2-I+2                                        N  13
 0011        2 F(J)=A(J)*F(J+1)+B(J)                            N  14
 0012          RETURN                                           N  15
 0013          END                                             N  16-
```

```
FORTRAN IV G LEVEL  20        SOURCE     DATE = 73046    11/33/59
 0001          SUBROUTINE SOURCE (J,I,CS,DS)                            
 0002          COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP, A  3
               1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IUI A  4
               2MEN,IHEAT,Z,TO,XSTEP/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD/B/B A  5
               3ETA,GAMA(2),TAUI,TAUE,AJI(2),AJE(2),INDI(2),INDE(2)/V/U(200),F(2,2 A  6
               4U0),R(200),RHO(200),OM(200),Y(200)/C/SC(200),AU(200),BU(200),CU(20 A  7
               50),A(2,200),B(2,200),C(2,200)/D/YR(200),UR(200),RR(200),HR(200),XM A  8
               6(200),PITOT(200),TEMP(200)/E/DSTAR(300),XRS(300),RWRS(300),COSAL(3 A  9
               .700)                                                              A 10
 0003          CS=SC(I)*(U(I+1)*U(I+1)-U(I)*U(I))/(OM(I+1)-OM(I))                  
 0004          CS=CS-SC(I-1)*(U(I)*U(I)-U(I-1)*U(I-1))/(OM(I)-OM(I-1))             
 0005          CS=(1.-1./PREF(J))*CS/(OM(I+1)-OM(I-1))                            
 0006          DS=0.                                                              
 0007          RETURN                                                            
 0008          END                                                              
```

```
FORTRAN IV G LEVEL  20        VEFF       DATE = 72286    22/52/18
 0001          SUBROUTINE VEFF (I,IP1,EMU)                              P   1
 0002          COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP, P   2
               1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IOI P   3
               2MEN,IHEAT,Z,TO,XSTEP/V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2 P   4
               300)/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD                      P   5
 0003          T=F(1,I)-U(I)**2/2.                                      P   6
 0004          TT=F(1,IP1)-U(IP1)**2/2.                                 P   7
 0005          T=(T+TT)/2.                                              P   8
 0006          EMU=T**ZETA/(REORS/SQRT(2.))                            P   9
 0007          RETURN                                                  P  10
 0008          END                                                     P  11-
```

65

```
FORTRAN IV G LEVEL  20              VISCO           DATE = 72286        22/52/18

  0001                    FUNCTION VISCO (I)                                        Q   1
  0002                    COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,   Q   2
                         1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI   Q   3
                         2MEN,IHEAT,Z,TO,XSTEP/V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2   Q   4
                         3OO)/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD                        U   5
  0003                    T=F(1,I)-U(I)**2/2.                                        Q   6
  0004                    VISCO=T**ZETA/(REORS/SQRT(2.))                             Q   7
  0005                    RETURN                                                     Q   8
  0006                    END                                                        Q   9-
```

```
FORTRAN IV G LEVEL  20              WALL            DATE = 72286        22/52/18

  0001                    SUBROUTINE WALL                                            R   1
  0002                    COMMON /GEN/ PEI,AMI,AME,DPDX,PREF(2),PR(2),P(2),DEN,AMU,XU,XD,XP,   R   2
                         1XL,DX,INTG,CSALFA,ALPHA,XR,REORS,GAM,ZETA,PPO,TWTO,YSTART,USUP,IDI   R   3
                         2MEN,IHEAT,Z,TO,XSTEP/V/U(200),F(2,200),R(200),RHO(200),OM(200),Y(2   R   4
                         3OO)/I/N,NP1,NP2,NP3,NEQ,NPH,KEX,KIN,KASE,KRAD/B/BETA,GAMA(2),TAUI,   R   5
                         4TAUE,AJI(2),AJE(2),INDI(2),INDE(2)                         R   6
            C             CALCULATION OF BETA FOR THE I BOUNDARY                     R   7
  0003      1             YI=.5*(Y(2)+Y(3))                                          R   8
  0004                    UI=.5*(U(2)+U(3))                                          R   9
  0005                    RH=.25*(3.*RHO(2)+RHO(3))                                  R  10
  0006                    RE=RH*UI*YI/VISCO(1)                                       R  11
  0007                    FP=DPDX*YI/(RH*UI*UI)                                      R  12
  0008                    AM=AMI/(RH*UI)                                             R  13
            C             FOR LAMINAR FLOW AND AM=0 (NEED DIFFERENT EXPRESSION IF F=0)   R  14
  0009                    S=1./RE-FP/2.                                              R  15
  0010                    BETA=RE*(S+FP+AM)                                          R  16
  0011                    TAUI=S*RH*UI*UI                                            R  17
  0012                    IF (NEQ.EQ.1) RETURN                                       R  18
            C             CALCULATION OF GAMA 'S FOR THE I BOUNDARY                  R  19
  0013                    DO 2 J=1,NPH                                               R  20
            C             FOR LAMINAR FLOW AND LIMITING ZERO AM                      R  21
  0014                    SF=1./(PR(J)*RE)                                           R  22
  0015                    GAMA(J)=RE*PR(J)*(SF+AM)                                   R  23
  0016                    IF (INDI(J).EQ.1) AJI(J)=SF*RH*UI*(2.*F(J,1)-F(J,2)-F(J,3))*.5   R  24
  0017      2             CONTINUE                                                   R  25
  0018                    RETURN                                                     R  26
  0019                    END                                                        R  27-
```

APPENDIX IV
INITIAL PRESSURE DISTRIBUTION COMPUTER CODE

```
1:  C      PROGRAM FOR INITIAL PRESSURE DISTRIBUTION
2:  .      READ (105,14) G,ALPHA,XR,XSTEP,AACT,AEFF,IDIMEN
3:  C      G=GAMMA (RATIO OF SPECIFIC HEATS)
4:  C      ALPHA IS DIVERGING NOZZLE WALL HALF ANGLE
5:  C      XR IS LONGITUDINAL RADIUS OF CURVATURE OF CONVERGING SECTION
6:  C      XSTEP IS STEP SIZE ALONG X COORDINATE (PERCENT OF LOCAL WALL RAD)
7:  C      AACT IS THE ACTUAL NOZZLE AREA RATIO
8:  C      AEFF IS THE EFFECTIVE NOZZLE AREA RATIO
9:  C      IDIMEN=0 FOR TWO-DIMENSIONAL NOZZLE AND 1 FOR AXISYMMETRIC NOZZLE
10: C      XWIG, ZWIG, AND RWIG ARE THE COORDINATES OF THE POINT WHERE THE
11: C      NOZZLE WALL SLOPES ARE MATCHED JUST DOWNSTREAM OF THE THROAT
12:        ALPHA=ALPHA*3.14159/180.
13:        NUM=0
14:        PI2=3.14159/2.
15:        COSALF=COSF(ALPHA)
16:        SINALF=SINF(ALPHA)
17:        ZWIG=XR*(1.+SINALF)
18:        XWIG=XR*(PI2+ALPHA)
19:        RWIG=1.+XR*(1.-COSALF)
20:        XTHOAT=XR*PI2
21:        REND=SQRTF(AACT)
22:        IF (IDIMEN.EQ.0) REND=AACT
23:        XEND=XWIG+(REND-RWIG)/SINALF
24:        CONST=(SQRTF(AACT)-SQRTF(AEFF))/(COSALF*(XEND-XWIG)**1.5)
25:        IF (IDIMEN.EQ.0) CONST=(AACT-AEFF)/(COSALF*(XEND-XTHOAT))
26:        ALPHA=ALPHA*180./3.14159
27:        IF (IDIMEN.EQ.0) WRITE(108,18)
28:        IF (IDIMEN.EQ.1) WRITE(108,19)
29:        WRITE (108,15) G,ALPHA,XR,XEND,XSTEP,AACT,AEFF
30:        C1=(G+1.)/2.
31:        C2=(G-1.)/2.
32:        C3=C1/(G-1.)
33:        C4=C1**C3
34:        C5=C4*C2*C3*2.
35:        FM=0.5
36:        X=0.0
37:        R=0.0
38:        DS=0.0
39: 1      CONTINUE
40:        XU=X
41:        X=X+XSTEP*R
42:        IF (X-XWIG) 2,2,4
43: 2      CONTINUE
44:        IF (.NOT.(XU.LT.XTHOAT.AND.X.GT.XTHOAT)) GO TO 3
45:        X=XTHOAT
46: 3      CONTINUE
47:        Z=XR*(1.0-COSF(X/XR))
48:        R=1.+XR*(1.-SINF(X/XR))
49:        RINV=R
50:        COSA=SINF(X/XR)
51:        GO TO 5
52: 4      Z=(X-XWIG)*COSALF+ZWIG
53:        R=(X-XWIG)*SINALF+RWIG
54:        COSA=COSALF
```

```
55:        DS=CONST*(X-XWIG)**1.5
56:        IF (IDIMEN.EQ.0) DS=CONST*(X-XWIG)
57:        RINV=R-DS*COSALF
58: 5      A=(1./RINV)**(1+IDIMEN)
59:        IF (A-1.0) 7,6,6
60: 6      A=1.0
61:        FM1=1.0
62:        FM=1.0
63:        B=1.0+C2
64:        GO TO 12
65: 7      IF (X-PI2*XR) 11,11,9
66: 8      B=(1.+C2*FM1**2)
67:        FM=FM1-(A*B**(C3+1.)-C4*FM1..)/(C5*FM1**2-C4*.1
68:        IF (ABS(FM-FM1)-0.00001) 12,11,11
69: 9      CONTINUE
70:        IF (FM-1.20) 10,11,11
71: 10     FM=1.20
72: 11     FM1=FM
73:        GO TO 8
74: 12     CONTINUE
75:        POP=B**(G/(1.-G))
76:        NUM=NUM+1
77:        Z=Z-XR
78:        WRITE (108,16) FM,PAP,Z,X,DS,R,RINV,NUM
79:        WRITE (106,17) X,POP,NUM
80:        IF (X-XEND) 1,1,13
81: 13     CONTINUE
82:        STOP
83: C
84: 14     FORMAT(6E10.0,I1)
85: 15     FORMAT(10X,'GAMMA =',F7.3//,10X,
86:       2'ALPHA =',F7.3//,10X,'XR     =',F7.3//,10X,'XEND   =',F7.3//,10X,'XSTEP
87:       3 =',F7.3//,10X,'AACT   =',F7.3//,10X,'AEFF   =',F7.3///, 8X,'M',11X,
88:       4'P/P0',12X,'Z',13X,'X',12X,'DS',12X,'RW',11X,'RINV',7X,'NUM'/)
89: 16     FORMAT(1P7E14.5,I6)
90: 17     FORMAT(2E12.6, 53X, I3)
91: 18     FORMAT(1H1,9X,    'ONE-DIMENSIONAL PERFECT GAS EXPANSION FOR A TWO-
92:       1DIMENSIONAL NOZZLE GEOMETRY WITH'//,10X,'THE ASSUMPTION THAT THE BO
93:       2UNDARY-LAYER DISPLACEMENT THICKNESS VARIES AS (X-XWIG)'//)
94: 19     FORMAT(1H1,9X,'ONE-DIMENSIONAL PERFECT GAS EXPANSION FOR AN AXISYM
95:       1METRIC NOZZLE GEOMETRY WITH THE'//,    10X,'ASSUMPTION THAT THE BO
96:       2UNDARY-LAYER DISPLACEMENT THICKNESS VARIES AS (X-XWIG)**3/2'//)
97:        END
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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Arnold Engineering Development Center Arnold Air Force Station, Tennessee 37389	UNCLASSIFIED
	2b GROUP N/A

3 REPORT TITLE

VISCOUS EFFECTS IN LOW-DENSITY NOZZLE FLOWS

4 DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
March 9 to November 30, 1972--Final Report

5 AUTHOR(S) *(First name, middle initial, last name)*

David L. Whitfield, ARO, Inc.

6 REPORT DATE	7a. TOTAL NO OF PAGES	7b. NO OF REFS
June 1973	77	18

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	AEDC-TR-73-52
c. Program Element 62302F	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	ARO-VKF-TR-73-8

10 DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11 SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
Available in DDC	AFRPL/DYSP Edwards Air Force Base California 93523

13 ABSTRACT

Viscous effects in low-density nozzle flows were investigated numerically, and comparisons were made with experimental data. The numerical method of Patankar and Spalding was modified to solve the internal laminar boundary-layer equations for two-dimensional flow or axisymmetric flow with or without transverse curvature. A listing is given of the computer code. Boundary-layer displacement thicknesses for typical nozzle geometries and flow conditions are presented. Solutions were obtained for specific conditions corresponsing to experimental data. The result is a relatively fast, simple to use numerical procedure, which is shown to give results in good agreement with experimental data.

DD FORM 1473 NOV 65

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
test facilities						
convergent divergent nozzles						
rocket nozzle						
scaling						
low density						
wind tunnels						
numerical analysis						

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